

## PHYSICS-BASED GENERATIVE DESIGN

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**Abstract:** We present a physics-based generative design approach to interactive form-finding. While form as a product of dynamic simulation has been explored previously, individual projects have been developed as singleton solutions. By identifying categories of computational characteristics, we present a novel unified model that generalizes existing simulations through a constraint-based approach. The potential of interactive form finding simulation is explored through exemplary studies: a conceptual approach to a fixed form that acts as a visualization of interacting forces, and a constraint-based model of the fabrication logic for a panelization system are examined. Implications of constraint-based simulation on future directions are discussed.

**Keywords:** Form finding, Dynamic simulation, Physics-based Design, Panelization.

**Résumé :** Dans ce papier on présente une approche générative basée sur la physique pour la conception des formes d'une manière interactive. Cette approche a été explorée précédemment mais seulement pour résoudre des problèmes isolés. En identifiant les catégories caractéristiques numériques, nous proposons un nouveau modèle unifié qui généralise les simulations courantes par une méthode à base de contraintes. Nous explorons la puissance de la conception interactive des formes par deux études concrètes : une approche conceptuelle qui visualise les forces interagissant sur une forme fixe, et une méthode à base de contraintes pour la construction logique d'un système de panneaux. Nous examinons les implications de la simulation à base de contraintes et les directions futures de recherche.

**Mots-clés:** forme recherchée, simulation dynamique, conception basée sur la physique, assemblage de panneaux.

### 1. Introduction

Physics-based generative design represents the synthesis of two characteristics in the digital design process. At the core, physics-based processes employ the simulation of complex natural phenomenon. Above this, we consider an active space of progressive formation and mutation, allowing highly plastic forms to evolve dynamically through an interaction of simulation components. Together, a number of these tools can be inter-related as part of the larger generative process to create new forms and compelling designs. Under such a definition, physics-based generative

design represents a paradigm shift from the traditional primacy of object to an exploratory approach of investigating interacting elements, interdependencies and systems. The integration of simulation opens up the possibilities for a more dynamic framework in the early stages of design.

Historically architects have explored animation tools as a generative method and as a source for comparative form exploration (Burry 2004). The idea of animation as simulation provides architects with an additional opportunity to explore new methods of design ideation by approaching design as a set of parameters responding to dynamic, material and variable contextual forces over time (Kolarevic 2003). The appeal of such a dynamic approach follows the growing interest of designers looking to nature as a source of novel processes and its equally novel outcomes.

Simulation has already allowed architects to pursue novel approaches to a design problem which has led to a growing number of investigations where simulation is an integral part of the form finding process (Oxman 2008). A survey of existing literature indicates an ongoing attempt to define terms and to find a meaningful role for simulation within the design process. We identify categories of the computational characteristics of key results and present a comprehensive unified computational model that captures these characteristics. Furthermore, we explore novel interactions of these characteristics that are only made possible by the use of a unified model.

## 2. Unified Solver

We present a new framework for dynamics simulations. The aim of this framework is to simulate the interactions between different objects and substances in a physics plausible manner. Traditionally solvers are designed to compute the motion of a particular type of object such as rigid bodies, cloth or rope. Combining effects such as a steel post in tension using a rope can be problematic as information has to be transferred between a rope solver and a rigid body solver when contact is made between the two objects. Instead, in our system, all objects are modeled as a *simplicial complex*: an assemblage of points, edges, triangles and tetrahedral (See Figure 1). These are all instances of a *k-simplex*, a mathematical generalization of the concept of a triangle (Alexandroff 1961). As every shape can be approximated to any desired precision with a simplicial complex, this generalization implicitly supports control over the quality of the simulation outcome.

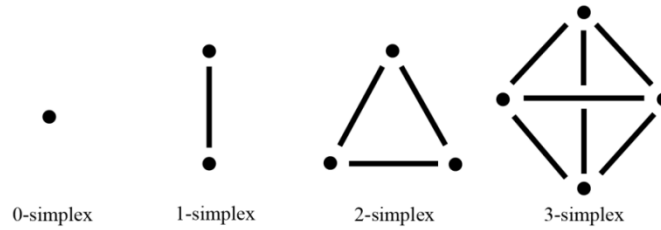


Figure 1.  $k$ -simplex shapes used in the unified solver (left to right): point, edge, triangle, and tetrahedron.

The dynamics of the framework are governed by a set of particles which correspond to the vertices of the simplicial complex under constraints (Arnold 1989). By modeling three simple constraints, namely edge length (stretch), angle between two edges (shear), and angle between two faces (bend) (see Figure 2), all meaningful deformations of 1-simplex, 2-simplex, and 3-simplex objects can be represented including torsion and shear.

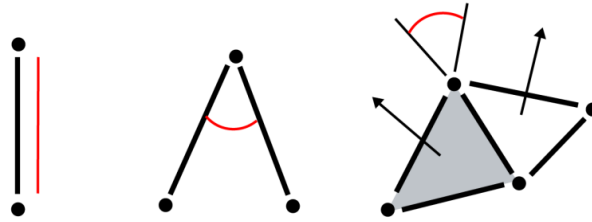


Figure 2. Three fundamental constraints (left to right): edge length, angle between two edges, and angle between two faces.

Material properties such as stretch, bend or shear are all formulated as constraints. For example, the stretch of a material is defined with respect to prescribed rest lengths. We choose this formulation as it is more stable than defining the stretch in terms of springs, for example. Springs are good at modeling bouncy objects but pose challenges when modeling stiff materials such as cloth. For very stiff materials, spring-based systems require very small time steps or fully implicit techniques which result in long simulation times. In our framework, we take the opposite approach where we start with hard links (constraints) and then allow them to be softened when more springy behavior is preferred. This results in faster and more stable simulations. Our solver falls in the category of symplectic integrators where velocities that resolve the constraints are computed implicitly while positions are updated explicitly (Hairer Lubich and Wanner 2002).

Another key feature of our framework is its ability to resolve collisions between objects and self-collisions for deformable objects. We perform the collision detection in space-time for better accuracy. This is necessary for a fast moving object which might be in a valid state at the beginning and at the

end of a simulation step but collides sometime midway. In this manner we guarantee that collisions are not missed. Our collision detection uses a fixed time step unlike solvers who treat collisions sequentially in order of their collision times. The latter approach can suffer from lockdowns and high computation times in the event of many collisions.

Collision handling can be seen as another constraint imposed on the system: no objects shall pass through each other. In general, a simplicial object in our system has to satisfy many different constraints at the same time. Sometimes these constraints can be in conflict such as a rubber band under tension between two poles. In this particular case the stretch constraint is battling the collision constraint. In most cases we want the collision to take precedence over the stretch constraint such that the rubber band is under tension. To better handle novel goals, the user can establish a preferred order of evaluation of the constraints. Rather than trying to solve each constraint one at the time, the solver interleaves them over a single time step. For each constraint, an importance weight is also assigned which determines how many times an attempt will be made to solve that constraint within each time step.

Complex emergent behavior occurs naturally. After adding air lift and drag constraints, for example, the flapping behavior of a piece of fabric emerges naturally due to these two constraints battling the stretch constraint. The air drag stretches or compresses the cloth which creates forces due to stretch. In this manner one can simulate complicated behaviors even with a very simple unidirectional wind model. Our general philosophy is to keep the basic solver steps as simple as possible and let complex behavior emerge from these simple components: complexity out of simplicity.

### **3. Physics-based Process**

Conceptual design is an open-ended process of discovery where the designer's imagination is at work to capture different design possibilities. Early design exploration is essentially a speculative process with its own dynamics, involving intuition and spontaneity (Aish 2005).

Drawing conventions, physical prototypes and CAD modeling are all essentially different modes of abstraction embodying various modes of design knowledge. These methods of abstraction are not neutral and they are adopted in relation to a particular design approach. For instance, associative modeling creates a conceptual design space based on a set of abstract geometrical rules and relationships.

A primary goal in the development of our approach has been to provide an open-ended framework that is not encumbered by geometric rules and relationships. The ability of this approach to parallel real world characteristics reduces the early needs for abstract procedural and

hierarchical development referred to as "designing the design" (Burry 2003). Physics-based simulation provides designers with intuitive metaphors for emergent form discovery by transforming the process of digital modeling to *digital empiricism*. This approach is not without precedents. Physically-based modeling and related optimization techniques as a means of geometric interaction has been a topic of interest in computer graphics for some time (Harada Witkin and Baraff 1995). For example, the use of constrained dynamics simulations for interactive geometric modeling was described and used by Gleicher and Witkin (1990) to support 2d drawing applications. Also, inspired by Antonio Gaudi, several simulation frameworks have been developed to support structural form-finding (Kilian 2004). Furthermore, in recent years, special effects technologies have been employed to facilitate dynamic sketching in the early stage of a design project (Mark 2007). However, each of these projects is a singleton solution. In this section, we present a number of previous simulation systems expressed as a simple set of constraints so that they can operate within a larger unified solver, opening up still more possible novel approaches to exploring the design space. We broadly polarize our classification of simulations as Collision-based or Equilibrium-based. Of course, when both classes are in play, we can achieve more complex emergent behaviors.

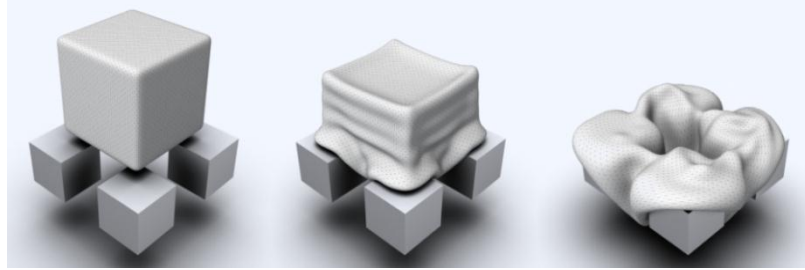
### 3.1. Collision

Contact between elements in the simulation is handled by calculating collision. The location of contact and the momentum transfer at the point of contact interact with material properties to deform and displace objects. *Draping*, *wrapping*, and *bounded growth* are prime examples of collision physics-based results.

#### 3.1.1. Drapery

The motif of drapery is one of the distinct characteristics of theory and practice in contemporary architecture. In the context of digital design, new advancements in digital processes have helped architects such as Frank Gehry to explore new forms of surface expression inspired by drapery (Allmer 2007). Gehry's design exploration is however set as an analog between the physical and digital model where physical models of draped surfaces are required to be digitized for further investigations. Simulation could provide an alternative to alleviate the physical interim process with virtual draping which could, perhaps, result in more varied outcomes.

In the example below (see Figure 3), a rounded cloth cube, with a high level of tessellation, is dropped under gravity onto four rigid cubes. The resulting deformations of the soft cube yield an organic structure that would be difficult to prototype physically.

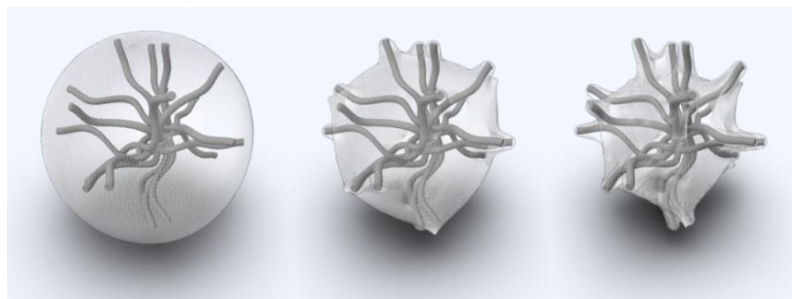


*Figure 3.* A soft cube draped over rigid cubes (left to right): initial condition, collision due to gravity and resulting deformation, and final shape.

### 3.1.2. *Wrapping*

Wrapping provides a conceptual model for skinning an intended object. In a way, wrapping is analogous to a fit fabric around a body of organized data. For instance, an arrangement of structural framing, or a collection of particles representing a flow of architectural programs, could be set up to create an envelope that wraps around them (Ophir 2008).

A shrink film can be made to shrink in one direction (unidirectional or mono-directional) or in both directions (bidirectional) along an initial surface that surrounds the structural frame. To achieve this effect in our solver, the rest length between vertices is set to zero or some progressively minimal value to gradually bring an initial surface into contact with the frame over time (see Figure 4). Collision of the surface with the frame will repel the surface and in time produce a shrink wrap. An air pressure constraint can also be used to aid the surface in better conforming to deep concavities in the frame by setting pressure inside the enclosing shrink surface to zero with normal pressure on the outside. Additionally, drastically different results can be explored by varying the shape and tessellation of the initial shrink surface.



*Figure 4.* Malleable surface conforming to an underlying rigid structure (left to right): initial condition, collision due to shrinkage and negative internal pressure.

### 3.1.3. Bounded Growth

Bounded growth is similar to the shrink-wrap process involving both an interior and an exterior shape. However, in this method, we reverse the relationship of these shapes and the surface area of the envelope is increased while contained within a boundary constraint.

To achieve this, a surface made up of cloth like material is placed inside a closed rigid bounding container. The rest length of the surface in a given direction is gradually increased until the surface begins to collide with the enclosing container and with itself. Over time, corrugations, bends and folds can occur to accommodate the increased surface area of the surface inside the container. This method could also be combined with some changes in material properties to allow sharp angular folds to develop (see Figure 5).

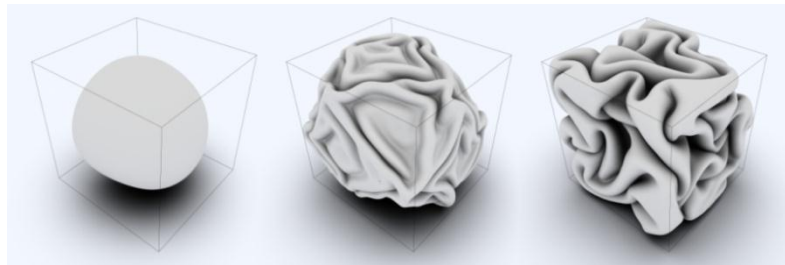


Figure 5. Growing surface bounded by an enclosure (left to right): initial condition, collision due to expansion and final shape.

## 3.2. Equilibrium

Equilibrium is the tendency for a system to achieve a stable balance between internal influences within that system. For instance, in designing fabric or grid shell structures designers aim to achieve an equilibrium position under the influence of loads by using computational methods such as dynamic relaxation. Relaxation is essentially a natural process that minimizes the potential energy in a system as that system tends towards equilibrium. The design of the British Museum Roof exemplifies this method by iteratively solving for the propagation of forces between all the nodes in the system (Williams 2001). Dynamic relaxation is typically applied when the overall form has already been fixed. A physics-based approach, however, opens up the possibility of using multiple sets of constraints with properties that would allow behaviors such as tension or compression to emerge as a form finding mechanism. Generally, the initial system is not in equilibrium before the simulation is started. After simulation begins many physical changes can be observed as elements in the system interact and change to achieve equilibrium. Observed changes in the system can also be captured during the process as starting points for other processes. The simulation can be run until

convergence or until a final state of equilibrium is achieved. In the case where a valid equilibrium state cannot be found, the simulation normally oscillates between different states in perpetuity.

During simulation, designers can also interact with the elements of the simulation changing the outcome and the possible states of transition. These changes may provide a vast number of design variations. Below we describe a number of key methods based on the notion of equilibrium.

### 3.2.1. *Gaudi Paradigm*

This paradigm refers to a classic method of structural form finding where form is defined through a translation of gravitational force. Antonio Gaudi's hanging chain models are the best known examples of using this scheme in which a building is modeled in tension under reverse gravity to define the form of the compression structure (under normal gravity). While this method has been previously explored, by making multiple physical models, a similar set up can be created as a real-time simulation (Kilian 2004).

By applying positional transform constraints to vertices of a planar surface and raising them to a given height during simulation, a tent like structure will emerge. Similarly, groups of nodes can be constrained to form creases of various shapes. As tension propagates through the fabric under motion, waves can form in the cloth until gravity and damping dissipate them allowing the system to reach equilibrium. In the example below (see Figure 6), a triangular piece of virtual cloth, that is pinned at the corners, stretches under the effect of reversed gravity. Varying material properties such as stretch, shear, rigidity and the bending between surface sub-elements can change the shape and nature of the resulting structure.

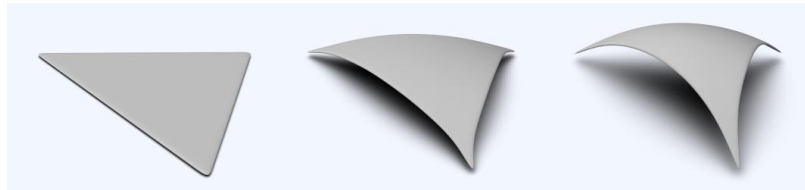


Figure 6. Gaudi Effect (left to right): negative gravity stretches a surface.

### 3.2.2. *Minimal Surface*

When a catenary curve is rotated about an axis, it creates a minimal surface area for the bounding circle called a catenoid. This can also be approximated using cloth and gravity under our solver. The structure shown in Figure 7 was created by intersecting, merging and smoothing two open cylinders. Positional constraints are added to the end annuli of the cylinders. The rest length is then scaled down for all elements of the material, putting the entire



surface in tension and allowing it to shrink. Sufficient stretch sub-steps are used in the simulation to avoid excessive non-uniform deformation.

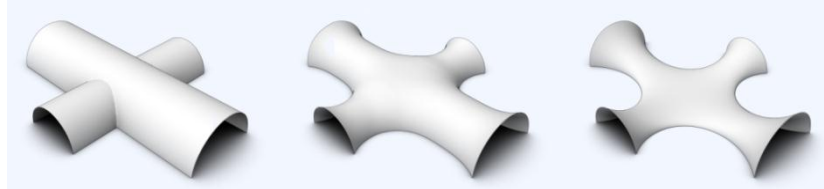


Figure 7. Surface minimization (left to right): rest length reduction.

#### 4. Study

Previously, we discussed collision and equilibrium separately. We now examine more complex scenarios where these classes interact and, furthermore, volumetric or logic-based constraints are involved in the simulation. A constraint-based conceptual design process can further be extended as the designer sees fit.

##### 4.1. Freeform Finding using Interacting Elements

In addition to each method described in previous sections, we can combine various methods to allow more complex behaviors to emerge. In the example below, a set of spheres with cut out areas are initially placed in a grid pattern. See Figure 8 (left). Using particle dynamics, a volumetric varying torsional force field is applied to the particles which are the nodes of simulated cloth. See Figure 8 (right). The simulation adds material properties and realistic deformation by colliding with a fixed ground plane. The interplay among all the internal material forces, collision and the torsional force field cause the entire structure to deform almost organically with dramatic effect. As the force field dissipates, the form settles to a stable state. These force fields could represent certain contextual conditions that are not strictly physical. Thus, simulation can be used not only to generate forms but also to produce the 'spatial coding of information' (Franken 2003).

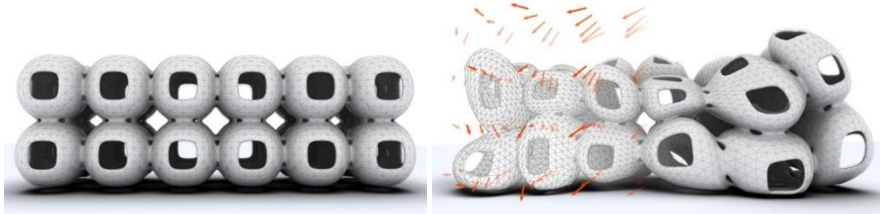


Figure 8. Interacting elements (left to right): deformation through an interplay between the internal material forces, collision and torsional force.

## 4.2. Embedding Fabrication Logic

Freeform architecture based on doubly curved surfaces is technically difficult and costly to directly realize as a physical artifact. Panelization is a technique to enable such a surface to be constructed from a series of smaller, simpler components. There is a considerable advantage if the panels are planar, since this enables the panels to be made from a standard material such as glass. Conventional planar panelization using triangular facets can be fitted to complex surfaces, but at each node six panel edges must be connected, which introduces additional fabrication complexity (Cutler and Whiting 2007). These connections can be simplified if quadrilateral panels are used. However it is non-trivial to define the set of planar quads (PQ) for a given surface, where each set of four adjacent panels meet at a common point (or structural node). In addition, because the sheet material (such as plywood) has a defined thickness, it is also important that the offset quads of each four adjacent planar quads also intersect at a common point (Cutler and Whiting 2007). Thus the full definition of the ‘implementation constraint’ is that the design surface has to be decomposable into Planar ‘Offset’ Quads (POQ).

In this example we explore a freeform surface design driven by a POQ mesh principle. While this class of surface has been previously explored as a mathematical optimization of a fixed surface (Pottmann, Schiffner and Wallner 2008), we are interested in exploring POQ meshes as a guiding principle of dynamic surface generation. Instead of approaching POQ meshes as an optimization problem, we embed their rationale within a flexible and iterative design process. Therefore, a freeform surface is defined as an emergent set of relationships among simpler components.

We begin the process by establishing the surface as a simulation of singular panels. By simulating the actual panels our system guarantees a constant offset within a numerical tolerance using collision between surfaces and constraints between points. As mentioned, material properties such as stretch, bend or shear are all formulated as constraints. Therefore, we apply the principles of the POQ mesh as constraints that define the inherent properties of panels. To assure planarity, each panel is essentially treated as a 3-simplex shape where the angle of two faces (bend) is minimized through cross bracing. See Figure 9 (a).

After setting the material property of the panels we establish a set of relationships among the panels in order to define the overall behavior of the surface system. These relationships are defined through two sets of constraints. One set of constraints welds all the panels together while allowing each panel to pivot around its border. See Figure 9 (b). After offsetting the surface, Figure 9 (c), the second set of constraints is applied as a distance constraint between the surface and its offset, thereby emulating

the thickness of the panels. The distance constraint allows the offset surface to slide while maintaining a constant offset value from the original surface. See Figure 9 (d). Once these connections have been established, we can manipulate the surface, either through pushing and pulling of nodes, or with other collision methods described earlier in this paper.

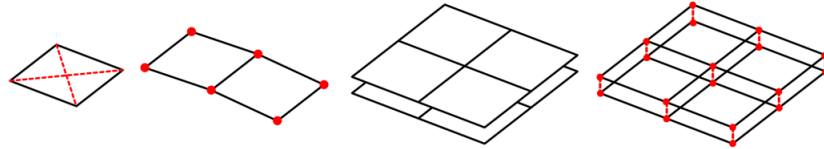


Figure 9. (left to right). (a) Planarity (build PQ face) by adding bend constraint. (b) Coincident vertex constraint to build "surface" from PQ face. (c) Creating an offset PQ mesh. (d) Creating POQ mesh by adding distance constraints.

The overall behavior of the surface can be characterized as a balance between precision and the degree of freedom. Unlike a typical simulation process which requires a well-defined model to converge at an optimum solution, we present a stable numerical model aiming at a more iterative progression but with fast results (see Figure 10). These results represent light-weight conceptual models that can be further refined in the later stages of design.

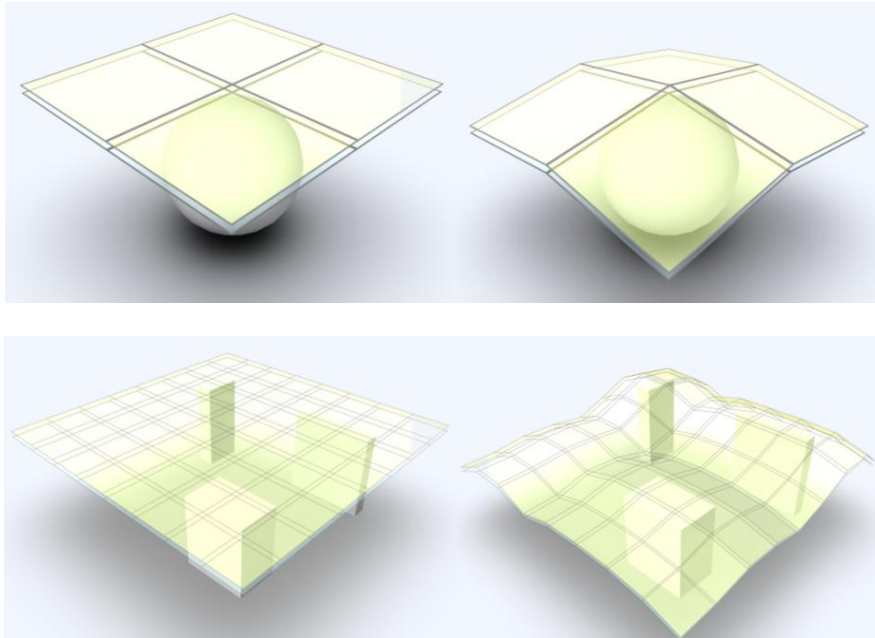


Figure 10. (Top to bottom): A basic POQ mesh interactively draped on a collision object. Using POQ mesh to interact with larger surfaces.

## 5. Discussion

As current digital processes facilitate an increasing formal complexity, rationalization strategies and methods are crucial to physical realization of complex forms (Schlueter and Bonwetsch 2008). The concept of "design rationalization" (Fischer 2005) is essentially a method of utilizing geometrical principles to achieve efficient assembly of different components. As Schlueter (2008) describes, design rationalization involves both pre-rationalization and post-rationalization. In pre-existing design methods either the subjectively arrived at building configuration is 'post-rationalized' into some simplified geometry allowing realistically constructible components, or a defined set of geometric constraints are established as a 'pre-rationalization' and the building form is constrained to conform to this geometry. The GLA building in London (Foster and Partners) is an example of a post-rationalized design method, where the original 'egg' form has been rationalized into PQ strips (Whitehead 2003). The Sage Performing Arts Centre in Gateshead (Foster and Partners) is an example of a pre-rationalized design method. In this case a decision was used to limit the surface to toroidal geometry, so as to standardize on a limited set of roof panels.

Our intention is to explore new approaches to design that were not previously available and which might offer ways to overcome some of the limitations of existing design methods. Our contention is that neither the post-rationalized nor the pre-rationalized approach addresses the challenge of resolving conflicting configurational and fabrication constraints. In the case of the post-rationalized method, a slight change to the overall configuration may result in a building form and geometry that is no longer suitable for the selected fabrication process, while in the case of the pre-rationalized method, the designer may feel unduly limited having to operate within the imposed geometry. Ideally the designer wants to experiment with changes in overall form and configuration (top-down) while at the same time exploring the consequence of different fabrication techniques (bottom-up).

Our approach uses a physics solver to unify all design constraints into a single model, and we have shown an instance of this approach by directly modeling the physics of the panel component. In contrast to post-rationalized and pre-rationalized method, we establish our fabrication constraint as *embedded-rationality* and the genesis of form exploration.

## 6. Conclusion & Future Work

In this paper we have presented a unified physics-solver as a comprehensive and generalized framework where problems (even ones that do not seem to be physical systems) can be re-expressed as a set of constraints resulting in

novel outcomes. By providing two different examples, we have distinguished between the use of a physics solver to model a hypothetical physical system for the purpose of creating some geometry that might be a ‘source of inspiration’, and the use of a physics solver to model a panelization system according to real world configurations and materials.

The example of the Planar Offset Quad (POQ) panels addresses a well known design problem, not with a specific algorithm but by using a physics engine which has much broader applicability. We contribute a general constraint-based framework that circumvents abstract geometric rationalization in the early stages of design. In working toward the development of more direct access to physical analogies for conceptual design, we turn to interaction design as the next logical step in this area. These are important challenges and additional research is required to define an interaction model for constraint-based simulation that is conducive to architectural design.

Given the flexibility and generality of our computational model we should consider the role of this method within the larger ecosystem of ‘generative’ and ‘analysis’ methods. While a distinct pipeline of methods, from concept to construction, can work quite well, the proposed method may naturally draw other design stages into early iterations of a project. By starting with a simulation-based process, the framework is already in place for migrating design concepts into design development, analysis and even other types of simulation. To achieve this we need to transform the role of simulation beyond pure analysis or isolated design cases. A comprehensive extensible simulation framework that ties together the entire design-to-production process is proposed. A unified solver also provides us with new possibilities for creating a bidirectional relationship between the physics engine and scripting control, parametric systems, or embedded analyses. We strongly feel that simulation for design will be an enabling technology for advancing the future of computer aided architectural design.

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