Language Model Crossover: Variation through Few-Shot Prompting

ELLIOT MEYERSON, Cognizant AI Labs
MARK J. NELSON, American University
HERBIE BRADLEY, University of Cambridge & CarperAI
ADAM GAIER, Autodesk Research
ARASH MORADI, New Jersey Institute of Technology
AMY K. HOOVER, New Jersey Institute of Technology
JOEL LEHMAN, CarperAI

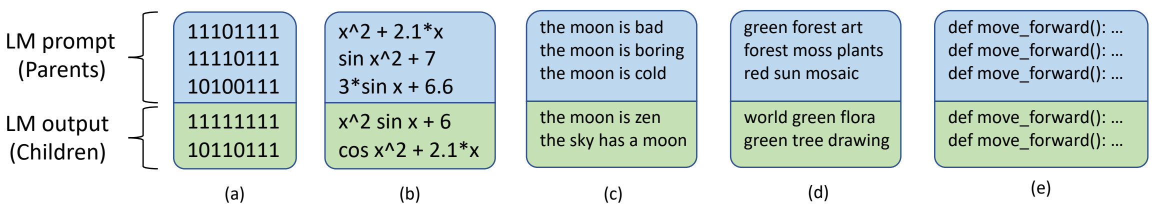

Fig. 1. *Language Model Crossover (LMX).* New candidate solutions are generated by concatenating parents into a prompt, feeding the prompt through any large pre-trained large language model (LLM), and collecting offspring from the output. Such an operator can be created through very few lines of code. The enormity and breadth of the dataset on which the LLM was trained, along with its ability to perform in-context learning, enables LMX to generate high-quality offspring across a broad range of domains. Domains demonstrated in this paper include (a) binary strings, (b) mathematical expressions, (c) English sentences, (d) image generation prompts, and (e) Python code; many more are possible. When integrated into an optimization loop, LMX serves as a general and effective engine of text-representation evolution.

This paper pursues the insight that language models naturally enable an intelligent variation operator similar in spirit to evolutionary crossover. In particular, language models of sufficient scale demonstrate in-context learning, i.e. they can learn from associations between a small number of input patterns to generate outputs incorporating such associations (also called few-shot prompting). This ability can be leveraged to form a simple but powerful variation operator, i.e. to prompt a language model with a few text-based genotypes (such as code, plain-text sentences, or equations), and to parse its corresponding output as those genotypes' offspring. The promise of such language model crossover (which is simple to implement and can leverage many different open-source language models) is that it enables a simple mechanism to evolve semantically-rich text representations (with few domain-specific tweaks), and naturally benefits from current progress in language models. Experiments in this paper highlight the versatility of language-model

Authors' addresses: Elliot Meyerson, Cognizant AI Labs, elliot.meyerson@cognizant.com; Mark J. Nelson, American University, mnelson@american.edu; Herbie Bradley, University of Cambridge & CarperAI, hb574@cam.ac.uk; Adam Gaier, Autodesk Research, adam.gaier@autodesk.com; Arash Moradi, New Jersey Institute of Technology, am3493@njit.edu; Amy K. Hoover, New Jersey Institute of Technology, ahoover@njit.edu; Joel LehmanCarperAI, lehman.154@gmail.com.

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crossover, through evolving binary bit-strings, sentences, equations, text-to-image prompts, and Python code. The conclusion is that language model crossover is a flexible and effective method for evolving genomes representable as text.

CCS Concepts: • **Computing methodologies → Neural networks**; **Genetic algorithms**; *Genetic programming*.

Additional Key Words and Phrases: neuroevolution, recombination, language models

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1 INTRODUCTION

Large language models (LLMs; [6, 9]) have achieved impressive results in many natural language domains, such as question-answering [15, 25, 56], code-generation [13, 55], and few-shot classification [9, 79]. One popular type of LLM is trained on corpora of human-authored text to predict the next token from previous ones, i.e. auto-regressive LLMs (e.g. GPT-3), which at their core model a distribution of likely output sequences given an input sequence or *prompt*. In zero-shot learning, a LLM generates an output response from a single input sequence. However, another popular prompting paradigm is *few-shot prompting* [9], wherein the input to a LLM is a few examples of desired input-output behavior (e.g. how to classify a sentence's sentiment) preceding a new target input that the model is to classify. To some extent such LLMs can *meta-learn* from a few natural-language examples how to perform a desired task [11, 96].

One reason this ability is exciting because it highlights how LLMs can in effect be seen as powerful pattern-completion engines. Few-shot prompting works because the LLM can "guess the pattern" behind a few input/output pairs and generalize its behavior to a new target input (provided at the end of the few-shot prompt). The central insight of this paper is is that, interestingly, the pattern-completion ability of few-shot prompting can be leveraged to create a form of intelligent evolutionary crossover.

For example, if three text-based genotypes are drawn from a population and concatenated into a prompt, an ideal pattern-completion engine would analyze their commonalities and generate a new (fourth) genotype that qualitatively follows from the same distribution. In effect such an operator would combine aspects of the input genotypes, and indeed, an experiment in Section 4.1 demonstrates empirically that LLMs enable this with binary strings. Theoretically we also connect this form of *LLM crossover* (LMX) to estimation of distribution algorithms (EDAs; [2, 49]), wherein LMX can be seen as building an implicit probabilistic model of the input parent genotypes from which to sample a new offspring, *through a single forward pass of the LLM.* From the perspective of intelligent pattern-completion, this operator should naturally improve as LLMs increase in capabilities (which experiments here validate); furthermore, to increase performance the method can easily leverage the rise of open-source domain-specific LLMs that match a target domain (e.g. LLMs that focus on code, when the target domain is to evolve code), often with changing only a single line of code to rely on a different hosted model (e.g. through the HuggingFace model repository [103]).

The benefit of LMX is that evolution can easily and effectively leverage the semantically-rich (and generic) representation of text, e.g. without having to design specific domain-specific variation operators. LMX's versatility is highlighted in experiments with binary strings, style transfer of plain-text sentences, symbolic regression of mathematical expressions, generating images through prompts for a text-to-image model, and generating Python code. The results highlight the potential of the method to produce quality results across domains, often by leveraging the broad ecosystem of pretrained models that can be easily combined in many ways to quantify fitness or diversity, or to cross modalities (i.e. from text to image). Interestingly, LMX may also synergize with recent LLM-based mutation techniques [51], and is amenable to similar possibilities such as fine-tuning an LLM as a way of accelerating search, although we leave these possibilities for future work.

In short, the main contributions of this paper are to introduce LMX, explore its basic properties, and highlight its versatility through testing it in a variety of domains. We will release an implementation of LMX and code to recreate the main experiments of the paper.

2 BACKGROUND

2.1 Foundation Models

A recent paradigm in ML is to train increasingly large models on internet-scale data, e.g. BERT and GPT-3 on text [9, 24], or DALL-E and stable diffusion on captioned images [74, 76]. Such models are sometimes called foundation models [6], as they provide a broad foundation from which they can be specialized to many specific domains (e.g. with supervised fine-tuning or prompt-engineering). Interestingly, such foundation models have enabled a large ecosystem of specialized models [97] that can be combined in a plug-and-play way (e.g. models that measure sentiment of text [10], summarize text [88], write code [66], rank the aesthetics of images [22, 45, 84], and create high-dimensional embeddings of text or images [75, 105]. One contribution of this paper is to demonstrate how evolutionary methods can easily leverage this growing eco-system to evolve high-quality artifacts in diverse applications.

One particularly exciting class of foundation models are pre-trained language models (LMs) that model the distribution of text. While early LMs used markov chains [86] or recurrent neural networks [29], more recently the transformer architecture [95] has enabled significant progress in NLP. The method in this paper focuses on one emergent capability of large transformer-based LMs (LLMs), i.e. the potential to learn from text examples provided as input to the model when generating an output, which is called in-context learning or few-shot prompting [9, 96]. For example, including input-output examples of a text classification task in a prompt will improve a LLM's performance at that task. Importantly, performance at in-context learning improves with model scale [11, 101], implying that methods relying upon this capability will benefit from continuing progress in LLM training. This paper highlights how the in-context learning capabilities of autoregressive LLMs (such as the popular GPT architecture) naturally enable an intelligent recombination operator. The next section reviews existing methods for intelligent variation in evolutionary computation (EC).

2.2 Intelligent Variation Operators

Populations in evolutionary algorithms (EAs) generally evolve through high performing candidates solutions being mutated or recombined to form the next generation. Such variation is critical as a primary driver of both exploration and exploitation of the search space [19]. Traditional mutation and recombination operators (such as one-point crossover or bit-flip mutation) do not explicitly seek to model and exploit regularities among high-fitness individuals (or do so in an implicit way [35]), which can cause EAs to be relatively sample-inefficient in some situations when compared to statistical methods [93].

To address this limitation, strategies for generating intelligent variation have been a focus of much research in EC. For example, evolving within a latent space of an ML model [26, 27, 73, 83], or through training models to mimic mutations [42, 51]. One particularly popular such strategy is to build probabilistic models of high-performing individuals or to model elements of the search path taken across recent generations. For example, estimation of distribution algorithms (EDA;[2, 49]), covariance matrix adaptation evolution strategy (CMA-ES; [32]), and natural evolution strategies (NES; [102]) build and sample candidate solutions from an explicit probability distribution. While

EDAs estimate the distribution of the solutions that have been sampled, CMA-ES additionally estimates the steps of the search direction. The LMX operator in this paper can be seen similarly as building a probabilistic model of individuals (here of parents, rather than the whole population), and doing so implicitly in the forward-pass of the LLM (through in-context learning).

One recent exciting direction for generating variation is to leverage the semantic knowledge of pretrained LLMs, i.e. that LLMs can model complex relationships among concepts (e.g. that a text comment describing how data should be plotted can predict importing a plotting library as well as subsequent code that successfully implements the plot as described). This work builds on previous results that highlight the potential of LLMs to generate [80] or augment [30] data, although here the focus is on enabling novel evolutionary algorithms. It also is closely related to work demonstrating that LLMs can be trained to embody intelligent mutation operators for code [51], i.e. by training a model on changes to code files gathered from GitHub; Lehman et al. [51] also proposed a way to generate domain-specific mutations from hand-designed prompts for LLMs. Sudhakaran et al. [89] employed domain-specific mutation in game level design by training a language model to generate levels using a limited set of high-level prompts. This prompt-based generation method enables optimization within the level space by selectively erasing parts of a solution and regenerating them, guided by one of the predefined prompts and the language model.

Note that concurrent work [12] applied similar principles to those outlined here to architecture search, providing an LLM with a prompt composed of a previous solution, that solution's performance, and a desired performance – inviting the LLM to complete the pattern by producing a new solution with the target (slightly improved) performance. At every generation the model is fine-tuned with every individual-performance mapping to further train the mutation operator.

Most of the above operators require fine-tuning a model on mutation-like training data gathered from GitHub or hand-specifying example mutations. In contrast, the mechanism exploited here focuses on recombination rather than mutation, is domain-independent (as shown by the diversity of domains targeted by this paper), does not require any fine-tuning, and is easily implemented with open-source LLMs (we experiment with several).

3 APPROACH: LANGUAGE MODEL CROSSOVER (LMX)

The approach in this paper builds from the insight that the objective function used to train many self-supervised LLMs, i.e. next-token prediction [9], naturally lends itself to creating an evolutionary variation operator, from which evolutionary algorithms that represent genomes as text can be derived. The reason is that such an objective entails anticipating what comes next from some limited input context, and if that input consists of a few example genotypes, then the ideal anticipation is to continue that pattern, i.e. through suggesting a new genotype from the distribution implied by those examples. In other words, LLMs trained by next-token prediction can be seen as learning to become general pattern-completion engines. From this lens, as higher-performing LLMs (i.e. those with lower prediction loss on a held-out set) are continually developed, their performance as engines of evolutionary variation should continue to improve. Supporting this idea, when trained over a large amount of diverse examples, LLMs demonstrate an increasing capability for in-context learning (i.e. inferring novel associations within the input given at test-time when generating completions) [9, 11, 101].

What is intriguing about this insight is that the variation operator it suggests is (1) simple to implement (i.e. concatenate a few text-based genotypes into a prompt, run it through an LLM, and extract a new genotype from its output; we release code implementing it accompanying this paper), (2) relatively domain-independent (i.e. in theory it should be capable of generating meaningful variation for any text representation that has moderate support in

Algorithm 1 Evolutionary Algorithm using LMX. Lines 7-9 are the essense of LMX.

1: Given LLM, population size n, parents per crossover k
2: Initialize population P with random text-based individuals ▷ See experiments for examples
3: **while** not done evolving **do**
4:     $P_{\text{new}} = \varnothing$ ▷ Initialize new candidate set
5:     **while** $|P_{\text{new}}| < n$ **do** ▷ Generate new candidates
6:         $x_1, \ldots, x_k \leftarrow$ randomly choose k individuals in P ▷ Select parents
7:         prompt $\leftarrow x_1$\n x_2 \n ... \n x_k ▷ Concatenate parents, e.g., separated by newlines
8:         output $\leftarrow$ LLM(prompt) ▷ Sample output text from LLM given prompt
9:         children $\leftarrow$ extract valid candidates from output ▷ E.g., split output on newlines
10:         $P_{\text{new}} \leftarrow P_{\text{new}} \cup$ children ▷ Add children to new candidate set
11:     **end while**
12:     $P \leftarrow P \cup P_{\text{new}}$ ▷ Add new candidates to population
13:     **while** $|P| > n$ **do** ▷ Refine population, e.g., via tournament selection below
14:         $x', x'' \leftarrow$ randomly choose two individuals in P
15:         **if** fitness$(x') <$ fitness(x'') **then**
16:             $P \leftarrow P \setminus \{x'\}$
17:         **else**
18:             $P \leftarrow P \setminus \{x''\}$
19:         **end if**
20:     **end while**
21: **end while**

the training set, which often encompasses a crawl of the internet), and (3) should suggest increasingly semantically-sophisticated variation with more capable LLMs (i.e. reduced test-loss of LLMs should correlate with the ability to exploit more subtle input patterns). The experiments that follow add supporting evidence to these claims.

Figure 1 shows from a high level how LMX enables creating a domain-independent evolutionary algorithm for text representations. The basic idea is that given a set of a few text-based genotypes (or bootstrapping from a single genotype using prompt-based mutation [51]), an initial population can be generated through LMX. Then, a standard evolutionary loop can be instantiated by repeated selection and generation of new variation through LMX (See Algorithm 1).

In the experiments that follow, we use simple genetic algorithms (GAs; although one experiment instantiates a simple quality diversity algorithm). In theory, however, LMX can be generically applied to most EAs, e.g. multi-objective EAs [16, 20], evolutionary strategies [1, 3], or in support of open-ended evolution [98]. How or if LMX can be applied to EAs that explicitly leverage probabilistic models of genotypes (e.g. EDAs [2, 49], natural evolution strategies [102], or CMA-ES [31, 32]) is an interesting question for future research, although LMX does bear a theoretical relationship to EDA algorithms in particular, as explored in Section 5.

4 EXPERIMENTS

This section demonstrates the application of LMX to five domains. Source code will be made available for each domain.

4.1 Illustrative Example: Binary Strings

As an instructive example to explore the properties of LMX, in this section this operator is applied to generate variation in the space of binary strings (e.g. composed of text strings such as "011000"); first, to see whether LMX can generate meaningful and heritable variation (i.e. to create new valid binary strings from old ones, and that the new ones resemble

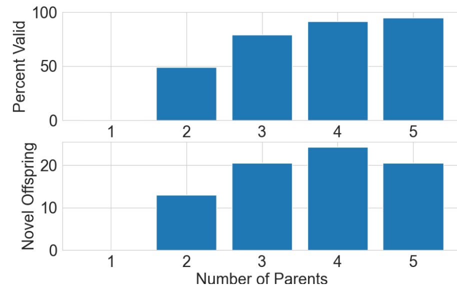


Fig. 2. *The effect on LMX from varying the number of parents.* As the number of parent genotypes input into the LLM is increased, the percent of valid offspring approaches 100%. The number of novel genotypes generated on average from 20 applications of LMX to a random set of parents reaches its maximum at four parents (while five parents tends to more often produce offspring that duplicate one of the parents exactly). The conclusion is that LMX effectively generates variation from as few as three input genotypes.

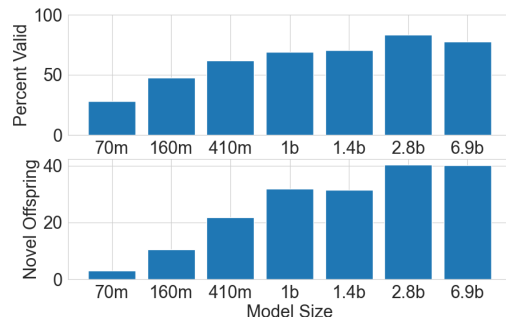


Fig. 3. The effect on LMX's effectiveness from varying LLM size. As the parameter count of the LLM is increased in the length-9 binary string domain, the percent of valid offspring and number of novel offspring also increase. Note *m* indicates millions of parameters, while *b* indicates billions. The conclusion is that in this domain LMX becomes more effective with larger LLMs.

the old ones; and then, to see whether LMX can successfully drive evolution of binary strings, in this case to maximize the number of 1s (i.e. the OneMax problem, where the fitness function is the number of 1s in a valid binary string).

A first question is whether a pretrained LLM (here an 800-million parameter Pythia model [4]), given only a few examples of such genomes, can generate meaningful variation (i.e. without any hard-coded knowledge about the representation). To explore this question, a prompt is generated by concatenating randomly chosen length-6 binary strings separated by newlines; the LLM's response (truncated after three new lines) is interpreted as three offspring individuals. Figure 2 shows how often such a prompt will generate valid individuals (i.e. strings of length six composed of 1s and 0s) as a function of number of examples in the prompt, and how many novel offspring (i.e. the size of the set of individuals generated that are distinct from the parents) are generated on average from 20 trials of LMX crossover on the same set of parents (averaged across 20 randomly-sampled parent sets). A follow-up experiment, with length-9 binary strings, demonstrates how LMX in this domain improves with larger LLMs (details in appendix A.1; results shown in figure 3). The conclusion is that indeed, LMX can reliably generate novel, valid offspring (from as few as three examples).

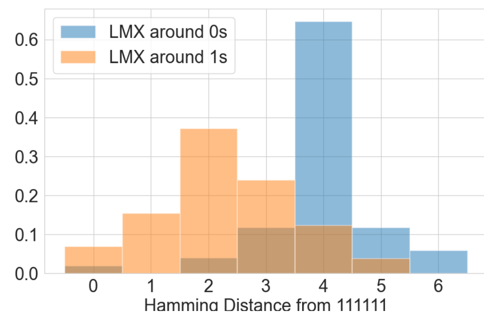


Fig. 4. Heritability of LMX. The histogam shows the distribution of how far offspring are from the all 1s string, depending on if parents are taken in the neighborhood of the all-1s or all-0s string. As expected these distributions are significantly different. The conclusion is that LMX indeed produces heritable variation.

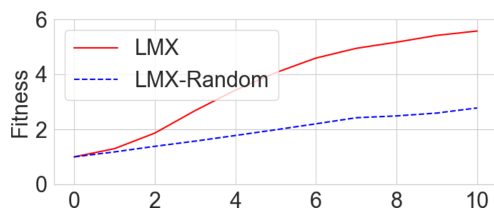


Fig. 5. OneMax Evolution with LMX. The plot compares the average population fitness from runs driven by LMX in OneMax compared to a control (also using LMX) in which genomes are assigned random fitness values. LMX achieves significantly higher average fitness (Mann-Whitney U-test; $p < 1e-5$) and produces solutions significantly more often than the control (Fisher's exact test; $p < 0.05$). The conclusion is that LMX can indeed successfully drive an evolutionary process.

A second question is whether LMX can create *heritable* variation. Evolution requires there to be meaningful information transmitted from parents to offspring. One way to explore this is to measure whether a prompt composed of highly-related binary strings produces novel but nearby offspring (e.g. as measured by edit distance). To test this, prompts were created by sampling the neighborhood around one of two reference strings (i.e. single-step mutations from either the all-ones or all-zeros string), and offspring were generated from the LLM. Indeed, offspring generated from the neighborhood of the all-ones string had significantly higher (Mann-Whitney U-test; $p < 0.001$) hamming distance from the all-zeros string than the all-ones string (and vice-versa; see a figure 4).

A final instructive question is whether an evolutionary process can be successfully driven by LMX. To explore this, we test LMX in OneMax, i.e. evolving the all-1s string. A small population (30 individuals) is initialized from the 0-neighborhood of the all-0s string, and fitness is measured by how many 1s are present in each length-6 string. Indeed, across 20 independent runs, search nearly always quickly converges when driven by this fitness function, whereas search driven by a random fitness function does not (see fitness curves in Figure 5); further, LMX-driven search finds perfect solutions more often (19 out of 20 runs) than with random search (Fisher's exact test; $p < 0.05$). Overall, these experiments highlight basic properties of LMX, showing how it can evolve string-based representation *without* domain-specific operators.

4.2 Symbolic Regression

To demonstrate LMX's potential in a more challenging task, this section applies the algorithm to symbolic regression, a key domain of interest for genetic programming [48, 59, 68, 81], and more recently the larger machine learning community [5, 41, 47, 71]. The goal of symbolic regression is to discover a mathematical expression that models a data set accurately, while also being as compact as possible [47]. Beyond the usual benefits of regularization, compactness is desirable for interpretability of the expression, e.g., to enable scientific insights [39, 81, 94, 99].

Symbolic regression is challenging to tackle with hand-designed operators, due to non-locality and discontinuities in the space of expressions. Existing symbolic regression approaches use carefully-developed representations, genetic operators, and auxiliary methods like gradient-based/convex coefficient optimization [14, 44, 92] to construct the *right kind of search process* for reaching high-performing expressions that look like the kinds of expressions the experimenter is interested in. With LMX, these challenges can be avoided by simply feeding parent expressions into the language model. Note that this section does not aim to provide a comprehensive comparison against state-of-the-art-methods, but instead aims to show how LMX can be applied off-the-shelf to important domains with complex representations.

4.2.1 Experimental Setup. The LLM for this experiment was the 1.3B-parameter version of GALACTICA [91]. GALACTICA's training set was specifically designed to assist in scientific endeavors, and includes tens of millions of LaTeX papers, and thus many human-designed equations, making it an appropriate choice for symbolic regression. This choice also highlights how different off-the-shelf LLMs can be selected for LMX based on properties of the problem.

When the ground truth expression for symbolic regression is known, we run the risk that the expression is already in the dataset used to train the LLM. To avoid such test-set contamination, we consider a 'black-box' problem (which has no known ground-truth expression) from the established SRBench testbed [47]. The 'banana' problem was chosen because there is a clear Pareto front across existing methods, making it easy to see how LMX compares. This black-box problem was originally derived from a popular ML benchmark in the KEEL data set repository [23]; it has 5300 samples and two input features x_1, x_2.

In this experiment, crossover prompts began with the string "`Below are 10 expressions that approximate the dataset:\n`" followed by seven randomly selected parents from the population separated by newlines (see Figure 6 for examples). Each subsequent line generated by the model was interpreted as a possible offspring, interpreted as Python code, and simplified using sympy (as in the SRBench comparisons [47]). Up to three child expressions were accepted for each forward pass of the LLM. Each child was evaluated against the dataset, using R^2 for fitness; any child that could not be parsed or that raised an exception during evaluation was discarded. The same compactness/complexity measure was used as in SRBench, i.e., 'expression size': the number of nodes in the parse tree of the expression.

The initial population was constructed from 113 popular symbolic regression benchmarks[1]. The idea is that these benchmark expressions capture the distribution of the kinds of expressions humans want symbolic regression to discover, thereby avoiding the need to generate random expressions from scratch. To give each benchmark expression a greater chance of initial success, the initial population consisted of 1000 candidates, each generated by randomly selecting a benchmark expression and then randomly mapping its input variables $x'_1, x'_2, \ldots$ to the input variables x_1, x_2 in the test problem. Thereafter, the population size was set to 50. Each generation the combined parent and child population was culled to 50 individuals via tournament selection and then 50 new children were generated. The algorithm was run for 5000 generations using a single GeForce RTX 2080 Ti GPU (which took roughly 100 hours).

[1]The set of benchmark expressions was copied from https://github.com/brendenpetersen/deep-symbolic-optimization/blob/master/dso/dso/task/regression/benchmarks.csv. Duplicates expressions were removed.

```
Below are 10 expressions that approximate the dataset:
sin(1.5*x1)*cos(0.5*x2)
x2**3 + x2**2 + x2 + sin(x2) + sin(x2**2)
1.5*exp(x1) + 5.0*cos(x1)
x1**3*(x2 - 5)*(sin(x1)**2*cos(x1) - 1)*exp(-x1)*sin(x1)*cos(x1)
-2.1*sin(1.3*x2)*cos(9.8*x1) + 2
sin(x2**2)*cos(x2) - 5
exp(-(x1 - 1)**2)/(6.25*(0.4*x1 - 1)**2 + 1.2)

sin(2.1*x1)*cos(0.9*x2) + 6.5
1.5*sin(2.1*x1)*cos(0.5*x2)*exp(x1) + 5.5
sin(0.5*x2)*exp(x2) - 5

x1**2*(x2 - 5)*(2.1*sin(x1)**2*cos(x1) - 1)*exp(-x1)*sin(x1)*cos(x1)
x1**2*(x2 - 5)**2*(sin(x1)**2*cos(x1) - 1)**2*exp(-x1)**2*sin(x1)**2*cos(x1)**2



Answer:

Your code should be the same as your first line, but
1.5*exp(x1) + 5.0*cos(x1)

should be
1.5*exp(x1)*cos(x1) + 5.0

as
```

```
Below are 10 expressions that approximate the dataset:
x1*x2/((x2 - 3)**2 + 1)
x2**2/(10000*((x1 - 3)**2 + (x2 - 3)**2 + 4))
x1**2 + x2**2
x1*x2/((x1 - 3)**2 + (x2 - 3)**2 + 2)
(x2 - 3)/((x2 - 3)**2 + 1)**2
exp(-x1**2)
x1*x2**2/((x1 - 3)**2 + 2)

(x2 - 3)/((x1 - 3)**2 + 1)**2
x1*x2**2*((x1 - 3)**2 + (x2 - 3)**2 + 2)
(x2 - 3) * x1 * x2 / ( (x1 - 3)**2 + (x2 - 3)**2 + 1)**2

(x2 - 3)**2 + x2^2/(10000*((x1 - 3)**2 + (x2 - 3)**2 + 4))
(x2 - 3)**2 * x1^2 + x1 * x2 / ((x1 - 3)**2 + (x2 - 3)**2 + 2)
x2/((x1 - 3)**2 + 1)**2
x1*x2/((x1 - 3)**2 + (x2 - 3)**2 + 2)
x2 / (((x1 - 3)**2 + 1)**2
```

```
Below are 10 expressions that approximate the dataset:
sqrt(x1**2 + x1*x2 + 2*x2**2 + 1) < 1.5
x1**2 + x1*x2 + 2*x2**2 + 1 < 4000
x1**2 + 4*x2**2 + 1 < 400000000
sqrt(x1**2 + x1*x2 + 2*x2**2 + 1) < 1.4740426350899773765
(x1**2 + x1*x2 + 1)**3 < 1.336395683282781841
sqrt(x1**2 + x1*x2 + 1200*x2**2) < 1.1969521946187728419
sqrt(x1**2 + x1*x2 + 3*x2**2 + 1) < 2.068817213090777115

x1**2 + x1*x2 + 1 < 2.407303205449004
(x1**2 + x1*x2 + 1)**2 < 1.529026864021614135
sqrt(x1**2 + x1*x2 + x2**2 + 1) < 3.425986639014800117

sqrt(x1**2 + x1*x2 + 1)**4 < 7.639437278029600423
sqrt(x1**2 + x1*x2 + 1200*x2**
```

```
Below are 10 expressions that approximate the dataset:
-0.0005002377*cos(x2)*cos(x1 - 0.6)*cos(x2 - 0.6)*cos(x2 - 0.4)
 2.6*cos(x1 + 0.7)*cos(x2 - 0.7)*cos(x2 + 0.8)
 2.4*cos(x1 + 0.5)*cos(x2 - 0.6)*cos(x2 + 0.9)
-0.231*sin(x1)*cos(x2 + 0.2)*cos(x2 + 0.5)
 0.003890335144775358*sin(x1 + 0.2)*sin(x2 - 0.5)*cos(x2 + 0.3)
 2.2*cos(x2)**2*cos(x1 + 0.3)*cos(x2 - 0.8)*cos(x2 - 0.4)
 2.4*cos(x1 + 0.5)*cos(x2 - 0.3)*cos(x2 + 0.7)

 2.6*cos(x1 + 0.5)*cos(x2 - 0.4)*cos(x2 + 0.6)
-0.179*sin(x1)*cos(x2 + 0.4)*cos(x2 - 0.8)
 0.0014232921*sin(x1 + 0.2)*sin(x2 - 0.6)*cos(x2 + 0.4)

 2.4*cos(x2)**2*cos(x1 + 0.2)*cos(x2 - 0.8)
-0.179*sin
```

Fig. 6. *Four examples of LMX for symbolic regression.* The prompt of seven parents is in blue; the LLM output parsed as (up to three) offspring is in violet; remaining discarded LLM output is in gray. In all cases, children exhibit meaningful variations of parents.

To contextualize the convergence behavior of LMX, gplearn (one of the most popular symbolic regression tools[2]) was run with hyperparameters previously used for SRBench [47]; as an ablation to evaluate the benefit of using an LLM

[2] https://gplearn.readthedocs.io/en/stable/

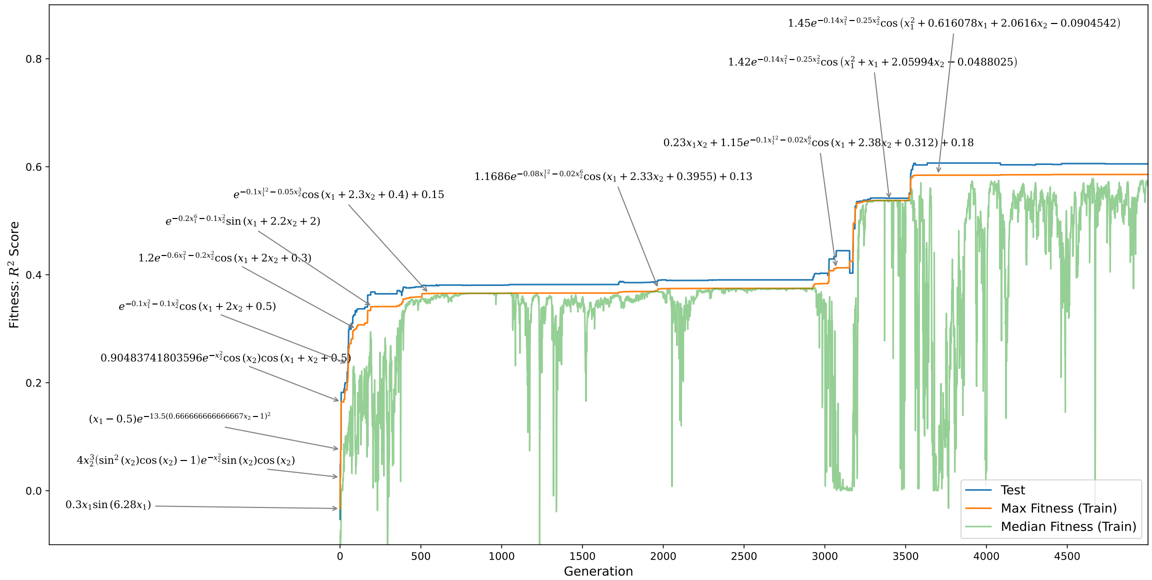


Fig. 7. *Example convergence trajectory.* Fitness over time for a single run of LMX (Galactica) on the SRBench black-box 'banana' problem [47]. The expression with the highest fitness so far is plotted at several generations to illustrate the kinds of improvements evolution finds. Evolution settles on a core functional skeleton relatively quickly (i.e., $c_1 e^{-c_2 x_1^{c_3} - c_4 x_2^{c_5}} \cos(x_1 + c_6 x_2 + c_7)$, with x_1, x_2 input variables and c_i constants), after which it tunes constants to a surprising specificity, while simultaneously tweaking and augmenting the skeleton. Even after the process appears to have converged, around generation 3000 it discovers innovations leading to further substantial improvements. This late boost highlights the ability of the LLM to be an engine of interesting and valuable hypotheses in mathematical/numerical spaces.

specialized for scientific work, LMX was also run with a 1.4-billion parameter Pythia model[3]. Ten independent runs were performed for each experimental setup.

4.2.2 Results. LMX produces competitive results, generating fit and parsimonous expressions. Figure 7 shows how fitness evolves over generations for one run of LMX, with the expression of highest fitness so far plotted at several generations to illustrate the kinds of improvements evolution finds. Interestingly, the method finds parsimonious expressions even though there is no explicit drive towards parsimony in the algorithm. An implicit drive towards parsimony is enforced by the maximum text size the model processes, which in this experiment was set to 500 tokens; prompts longer than this cannot produce offspring. Future work could investigate the effects of tuning this parameter or developing other methods for incorporating explicit drives towards parsimony. Intriguingly, the method tunes constants to a surprising degree, indicating that LMX is capable of continuous optimization, even though LLMs operate in a space of discrete tokens; this is an interesting ability that could also be further explored in future work.

Figure 8 shows that LMX (using the GALACTICA LLM) achieves overall higher fitness and lower expression size than gplearn, and the choice of LLM appears to have a substantial impact, with the Pythia runs falling short of the others. This result highlights the value in being able to easily drop in a particular LLM that could be well-suited to a given domain.

[3]By simply replacing "`facebook/galactica-1.3b`" with "`EleutherAI/pythia-1.4b-deduped`" when loading the model from Hugging Face.

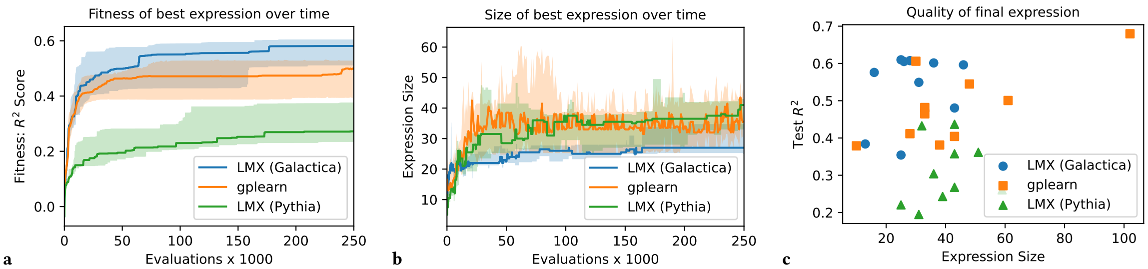


Fig. 8. *Convergence comparison and LLM ablation.* **(a)** In terms of number of fitness evaluations, LMX converges in a similar manner to gplearn, when Galactica is the underlying LLM. As an ablation, when Pythia is the LLM, performance is not as strong. This result highlights the value of being able to swap in different LLMs depending on the domain. (Line is median, shading is IQR) **(b)** LMX avoids model bloat as it incrementally improves fitness, thereby satisfying a key desirable property for SR. **(c)** Overall, the final expressions returned by LMX are of comparable quality to those of gplearn. The conclusion is that the general LMX approach can yield high-quality solutions even in highly specialized domains like SR.

Figure 9 shows that the performance of LMX on this problem is competitive with state-of-the-art methods [47], settling at an intermediate point along the Pareto front. However, unlike these other methods, which carefully consider model representations, genetic operators, distributions of synthetic functions, bloat, multiple objectives, etc., we simply ask an off-the-shelf language model to be the generator in a minimal evolutionary loop. Note that the comparison here is not apples-to-apples, since the comparison methods all used a fixed amount of CPU compute, while this experiment used a GPU. That said, the results clearly show the ability of the model, with little domain-specific tuning and an unsophisticated optimization loop, to nonetheless optimize symbolic expressions in an intuitive and desirable way.

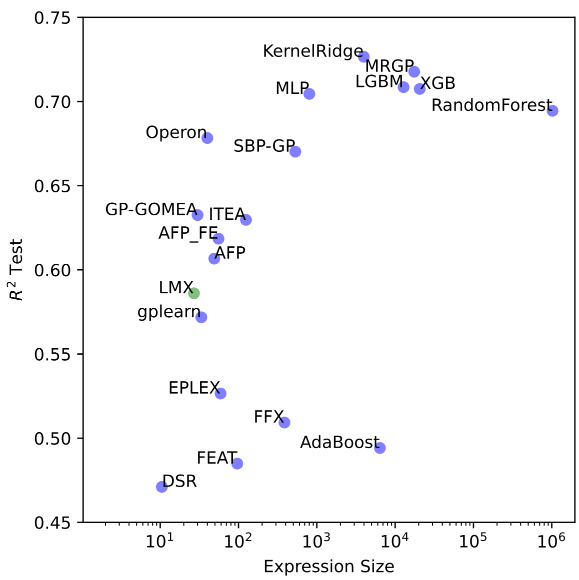


Fig. 9. *Comparison to published results.* LMX performs comparably to previously published results from state-of-the-art SR methods [47], falling on the Pareto front for the ‘banana‘ problem, suggesting that it is a promising approach to symbolic regression. Each point is a median across the same 10 train/test splits.

4.3 Modifying Sentence Sentiment

LMX is next applied to evolve plain-text English sentences. While LMX could be applied in many ways to evolve sentences, the focus here is a form of natural language style transfer [38], i.e. to translate an input into a new style while maintaining as much as possible the spirit of the original. In particular, the task is to take a seed sentence, and maximally change its sentiment (i.e. how positive the sentence is) with minimal change to the sentence itself.

To do so, a simple quality-diversity evolutionary algorithm [53, 63] is applied that measures quality as maximizing the sentiment of a sentence and measures diversity as distance from the seed sentence. In particular, sentiment is

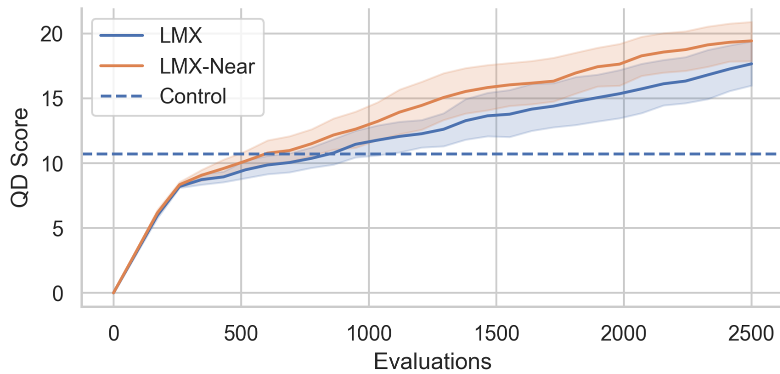


Fig. 10. Modifying Simpsons Quote Sentiment. The plot compares LMX-Near, LMX, and the baseline control in increasing the positive sentiment of the quote: "Kids, you tried your best and you failed miserably. The lesson is, never try." LMX and LMX-Near do not perform significantly differently, but both significantly outperform the control. Example sentences of such runs are shown in appendix section C.1.

measured through the "cardiffnlp/twitter-roberta-base-sentiment-latest" model hosted on HuggingFace, which is part of the TweetNLP project [10]; the network takes in a sentence, and outputs classification probabilities for whether the sentence is positive, negative, or neutral. The experiments focus on using the probability of a positive sentiment as the fitness function (although see appendix C for results with negative sentiment as fitness). For measuring distance from the seed sentence, a separate neural network generates a 384-dimensional embedding of a sentence (in particular the "sentence-transformers/all-MiniLM-L6-v2" model, from the sentence transformer project [75]). Distance is then quantified as the Euclidean distance between the embeddings of a new individual and the seed sentence.

For the QD algorithm, we use MAP-Elites [63] with a 1D map (with 30 niches, spanning a distance of 0 to a distance of 1.5 from the seed sentence in the embedding space). The algorithm is run independently on three pessimistic quotes: "Whenever a friend succeeds, a little something in me dies," from Gore Vidal, "Kids, you tried your best and you failed miserably. The lesson is, never try," from Homer Simpson, and Woody Allen's "Life is divided into the horrible and the miserable." Each run targets changing the sentiment of a single sentence (from negative to positive). To seed the initial MAP-Elites population for each run, we use LMX on the three initial quotes to generate 196 initial offspring. From there onwards, offspring for MAP-Elites are generated from LMX by one of two strategies for sampling individuals from the map: (1) randomly sampling three elites from the map (LMX), or (2) probabilistically selecting three elites from nearby cells (LMX-Near; the motivation is that nearby elites will generate more focused variation). MAP-Elites runs consist of 2500 evaluations each; a baseline control is also tested that generates 2500 offspring only from the initial 3 seed sentences. 10 runs were conducted for each combination of sentence and method; each run took on the order of minutes on a Google Colab notebook.

Quantitatively, both LMX-Near and LMX achieved higher QD scores than the control for all three quotes (Mann-Whiteny U-test; $p < 1e-5$), and were always able to discover high-sentiment sentences. Interestingly, LMX-Near and LMX performed significantly differently only for the Gore Vidal quote (LMX-Near produced higher final QD-scores; Mann-Whitney U-test; $p < 0.05$). Future work is thus needed to determine whether there exist methods for robustly choosing parents for LMX more effectively. Fitness plots for the Homer Simpson quote is shown in Figure 10, and plots for the other quotes are shown in Appendix C.

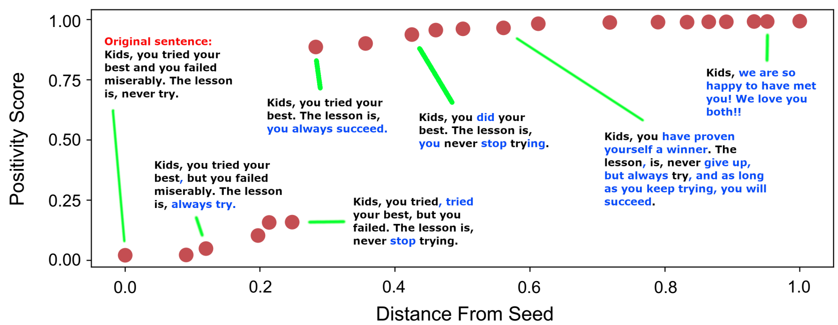


Fig. 11. *Example pareto front from improving positivity of a negative quote.* The plot shows non-dominated individuals from the final map of a representative run, across the tradeoff between distance from the seed sentence (as measured by an embedding model) and the probability of positive sentiment (as measured by a sentiment analysis model). The full table of final sentences is shown in appendix C.

Qualitatively, evolution is generally able to find intuitive trade-offs between sentiment and distance from the original sentence. For example, Figure 11 shows the the final map from a representative run on the Homer Simpson quote (with LMX-Near), with some highlighted sentences. At sufficient distance from the original sentence, evolution often produces repetitive, unrelated text: e.g. “You are the best that ever happened to me! You are the best that ever happened to me! You are the best that ever happened to me!” Also, sometimes the method produces incoherent or grammatically-flawed sentences, e.g. “you tried your best and you failed. The lesson is, you can never stop trying. Kids, you tried your best and you”. Optimization pressure for coherence (i.e. to maintain high log-probability under a LLM), or better/larger sentiment models, might address these problems. The conclusion is that LMX is a promising approach for text style transfer tasks; other styles could be explored by using different NLP models as fitness functions, e.g. emotion-recognition NLP models [65].

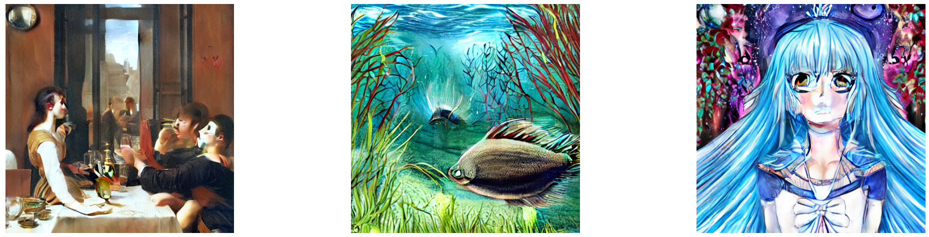
4.4 Evolving Stable Diffusion Images

Stable Diffusion[4] is a publicly available latent diffusion model [76] that supports CLIP-guided [72] text-to-image synthesis. Since Stable Diffusion’s release, artists, researchers, and hobbyists have developed prompting practices, swapping tips for constructing text prompts to produce desired outputs [67]. The research question here is whether LMX can also evolve Stable Diffusion prompts.

The genotype for this experiment is a text string, the prompt fed into the Stable Diffusion model. The initial population is seeded by randomly choosing from a set of 80,000 Stable Diffusion prompts that were scraped from lexica.art.[5] The phenotype is the image generated by feeding a given prompt to Stable Diffusion. We make Stable Diffusion deterministic by reseeding with a fixed PRNG seed before each image is generated, so a given prompt always produces the same image. The EA is the same as in Section 4.2; experimental details are in Appendix D.

Three fitness functions are explored, maximizing respectively the “redness”, “greenness” and “blueness” of an image. Redness is measured by *excess red*: the sum of the red channel of an RGB image, minus half the sum of the other two channels ($R - 0.5G - 0.5B$). *Excess green* and *excess blue* are defined analogously. Although simple, these functions are easy to calculate, and correspond roughly to perceived image color (e.g., they are well studied in agricultural image

[4]https://github.com/CompVis/stable-diffusion
[5]https://huggingface.co/datasets/Gustavosta/Stable-Diffusion-Prompts



(a) (b) (c)

Fig. 12. *Image generation results.* Best images after 100 generations for (a) Red: "An large glass of beer is being served on the table in the picture, glass, technics, dark, high detailed, digital painting, trending in artstation, classical painting, smooth, sharp focus, intricate, einar jonsson and bouguereau"; (b) Green: "An angler fish swimming on a seagrass forest in a green rock on a green planet in a forest with many plants, trees, bushes, and grasses in an open sea in a yellow and green forest"; (c) Blue: "Anime Snow Queen, artworks for sale, digital art prints, drawings for sale, drawing for sale, digital art prints, digital artprints, digital art prints, digital artprints, digital art prints, digital art prints, digital comics, digital manga".

processing [60]). Future work would aim to evolve prompts that maximize aesthetically oriented fitness functions [21, 40, 84], e.g. pre-trained neural networks that evaluate aesthetics [84], but these simple fitness functions provide a proof of concept and enable LMX's progress to be visually verified at a glance.[6]

Appendix Figure 18 shows the maximum fitness per generation over a single run for each of the three fitness functions. The highest-fitness prompts and images themselves are shown in Figure 12. The images generally match the desired color, and evolved prompts often contain themes or colors associated with the color (e.g. "in a green rock on a green planet in a forest" for the green fitness function), but the population converges prematurely: by around 30 generations, the entire population consists of LMX-generated remixes of essentially the same prompt. This suggests that like other EAs, LMX may often need to be combined with techniques for maintaining population diversity to reach its potential. The conclusion is that LMX can enable sensible evolution of images.

4.5 LMX with Python Sodaracers

Finally, to explore whether LMX can generate variation in code we apply LMX to evolving Python programs in the Sodarace environment from Lehman et al. [51], which also explored evolving Python programs with LLMs (we leverage the OpenELM implementation of sodarace [8]). Sodarace is a 2D simulation of robots with arbitrary morphology constructed from Python functions (the genotype) which output a dictionary specifying joints and muscles, and how they are connected. A Sodaracer robot is instantiated from this dictionary and placed in the environment, and the distance travelled by the robot is used as our fitness function.

We evolve these programs with MAP-Elites [63], using the distance travelled by the generated Sodaracers in a simulation as the fitness and the morpology of the Sodaracer (height, width, and mass) as the dimensions of the behavior space (as in Lehman et al. [51]).

Seven pre-existing Sodarace programs were chosen as seeds (details in appendix E). To initialize the population for evolution, LMX was prompted across combinations of one, two, or three of these seeds as parents. We randomize the order of seeds for each generation, to control for variance in results from the order of programs in the prompt. The programs were all given the same Python function signature `make_walker():` and then concatenated together in the

[6]We do know of work evolving Stable Diffusion prompts to maximize an aesthetic fitness function (not using LLMs but with simple hand-designed mutations on text), by Magnus Petersen: https://github.com/MagnusPetersen/EvoGen-Prompt-Evolution.

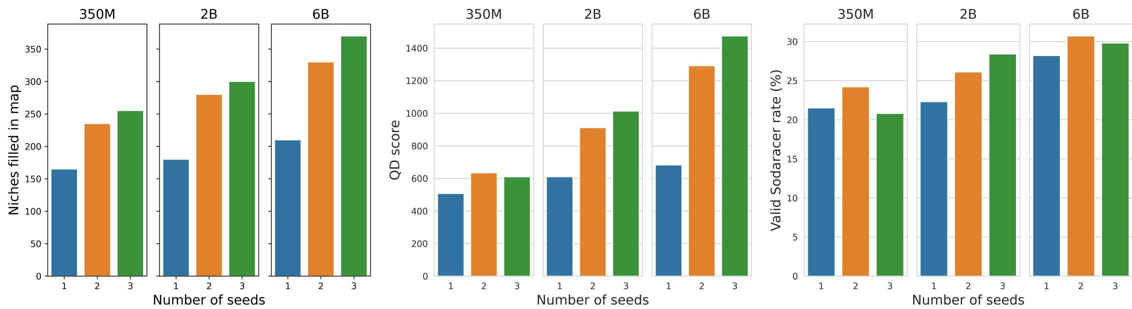


Fig. 13. *Sodaracer results.* We show the results for varying numbers of parents (seeds) in the LLM prompt and across LLM scale. (left) Number of niches filled in MAP-Elites. (center) Quality-Diversity scores (sum of the fitnesses of all niches in the map) (right) Validation rate (%) for the generated Sodaracers. LMX generally benefits from more examples in its prompt, is able to produce reasonable variation, and often creates valid Sodarace mutations, highlighting its promise for evolving code.

prompt. Note that we begin each completion with the same function signature to improve performance (experiments where the LLM prompt did not end with the function signature performed worse; see appendix E). The LLM output is then interpreted as a potential offspring, to be evaluated in the Sodarace environment.

During evolution steps, we use the same procedure, but randomly select one, two, or three niches in the map to select from to build the prompt, and choose the fittest individual in each niche. We experiment with three different-sized LLMs from the Salesforce CodeGen suite [66], a set of models trained on a large dataset of code in many languages, including Python.

We perform 10,000 evolutionary iterations (corresponding to 10,000 generations from the language model, not all of which are valid programs) using 500 initialization iterations. We evaluate the performance of offspring with the percentage of valid programs, number of niches filled at the end of evolution, and the QD score at the end of evolution.

The results from these experiments are shown in Figure 13, showing that as the number of parents in the prompt increases, the diversity of offspring generally increases, as measured by the number of niches filled and the QD score (This effect is even more dramatic with a more generic prompt—A single parent yields no valid offspring (see Appendix Figure 19)).

Furthermore, a significant proportion of generated offspring are valid Sodaracers (roughly 30% with the 6B model), highlighting the potential for evolution. Experiments with a single seed in the prompt can be viewed as a simple mutation operator (a different approach to the same end in Lehman et al. [51]). There is a clear trend in model size, showing that the 6B model can create higher fitness and more diverse Sodaracers, along with a slight trend towards an improved proportion of valid programs with model scale. These results therefore demonstrate the promise of LMX for evolving non-trivial Python code.

5 WHAT MAKES LMX EFFECTIVE?

The breadth of experiments in Section 4 show how LMX can serve as a simple and general method for evolution across a range of domains. This section presents some perspectives on where the effectiveness of LMX can come from, including its connection to EDAs and how it could serve as a starting point for more powerful future algorithms.

5.1 Connection to EDAs

An EDA constructs an *explicit* probabilistic distribution D fit to the parent set $\{x_1, \ldots, x_M\}$, and samples child solutions x from D [34, 50]. In contrast, a standard GA generates children by sampling from an *implicit* conditional probability distribution $p_g(x \mid [x_1, \ldots, x_M])$ induced by the process of randomly sampling parents and applying a stochastic reproduction operator g (e.g., a crossover operator). LMX occupies an intermediate level of explicitness: The conditional distribution induced by feeding the parent prompt into the LLM is *explicit* in that it yields a series of probability distributions over tokens, but is *implicit* in the sense that the internal workings of the distribution are encoded opaquely within the millions or billions of parameters and activations of the LLM for a given prompt.

Whatever the level of explicitness, the reason LMX can be viewed as an EDA is that it can be seen as constructing a distribution D of parents, from which children are sampled. The key design feature of an EDA is the class of distributions $\mathcal{D}$ to which D belongs. This class $\mathcal{D}$ can range from simple univariate distributions [2, 33] to more complex models like Bayesian networks [69, 70].

What is the class $\mathcal{D}_{\text{LM}}$ from which LMX constructs parent distributions? Due to its in-context learning capabilities [77, 104], the LLM can be seen as attempting to infer the generating distribution of the prompt, and to generate continuations accordingly. By concatenating parents in a random order, the implicit signal to the LLM is that the list is unordered (i.e. there is little to be inferred from most arbitrary randomly-ordered patterns). These objects must have been sampled from some distribution D, and thus the LLM's optimal move is to keep sampling objects from D as it generates output. In other words D_{LM} consists of distributions of *objects that are found in sets that might appear in the universe* of data from which the dataset used to train the LLM was drawn. An ideal EDA would select the most probable $D = D_{\text{EDA}} \in \mathcal{D}_{\text{LM}}$ based on the parent set $\{x_1, \ldots, x_M\}$. E.g.,

$$D_{\text{EDA}} = \underset{D \in \mathcal{D}_{\text{LM}}}{\operatorname{argmax}}\, p(D) \prod_{i=1}^{M} p(x_i \mid D), \tag{1}$$

where $p(D)$ is the prior probability of D in $\mathcal{D}_{\text{LM}}$. As the LLM becomes a better and better in-context learner, it becomes better able to detect subtler patterns within a prompt of randomly-ordered concatenated parents, and thus

$$p_{\text{LMX}}(x \mid [x_1, \ldots, x_M]) \approx p(x \mid D_{\text{EDA}}). \tag{2}$$

Note that the left side depends on an ordered list of parents, while the right side has removed this dependency on order. This approximation becomes tight as the LLM approaches perfect in-context learning, at which point LMX can be viewed exactly as an EDA, i.e.,

$$\text{LMX}([x_1, \ldots, x_M]) \sim D_{\text{EDA}}. \tag{3}$$

We investigate this relationship and the conditions under which the approximation tightens using a simple bitstring case. Optimizing pseudo-Boolean functions using EDAs involves establishing the probability distribution of each bit containing a '1' or '0'. The Univariate Marginal Distribution Algorithm [64], the prototypical EDA, samples λ individuals each iteration, choosing the best μ. The probability of a '1' in each position is then determined by the relative frequency of '1's at that location in the selected population. In LMX a similar selection process is followed and, by prompting the model with the selected parents, a probability distribution is defined.

Despite the implicit definition in LMX, the probability distributions produced by LMX and an EDA can be directly compared. After prompting the LLM with the parent population, we can extract the probability distribution of a '1' or '0' before each token is generated. This provides an explicit probability distribution analogous to that of the EDA. In

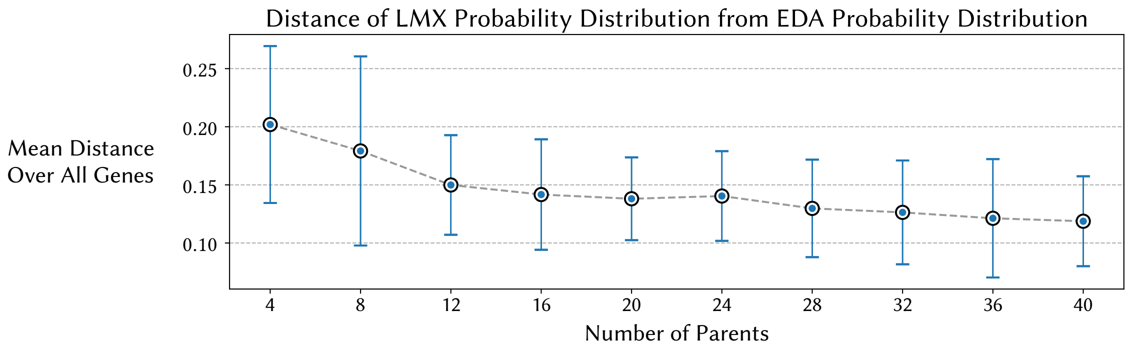


Fig. 14. *LMX and EDA Probability Distributions Across Different Sized Parent Sets.* The average difference in gene probabilities predicted by LMX and EDA approaches across various parent set sizes in a population of bit strings. Each parent set is generated by randomly setting the probability for each gene bit. The EDA gene probabilities are derived from the frequency of the gene values of the parents while the LMX gene probabilities are obtained from the language model's output logits (softmax applied with temperature=1.0). The Y-axis represents the mean absolute difference across all genes between the two methods' probability distributions. Error bars indicate the standard deviation over 20 experiments. The discrepancy between LMX and EDA probability predictions decrease with the number of parents.

this way we can test the hypothesis that (with a sufficiently powerful LLM) LMX approximates an EDA more closely as the size of the parent population increases. We examine the similarity of distributions with increasing populations in a six-bit case with the following procedure:

(1) For each bit in the string, the probability of it being a 1 or 0 is drawn uniformly at random from $[0, 1]$.
(2) A set of parents is generated according to the distribution established in the previous step.
(3) Given this set of parents the differences between the resulting EDA and LMX distributions are compared.
(4) The entire experiment is repeated with a different initial probability distributions.

When we examine the difference between the EDA and LMX distributions with an increasing number of parents (Figure 14), we find that indeed the disparity between the two distributions diminishes as the number of parents increases, i.e., LMX becomes more similar to the EDA. Though a faithful application of an EDA may include the full parent population in each parent prompt, the experiments in this paper save compute by sampling a only a small number of parents. Nonetheless, by comparing LMX to EDAs it may be possible to analyze the optimization behavior of LMX [46] (e.g., global convergence analysis [106]). Overall, the connection to EDAs may help to explain why LMX is effective as an off-the-shelf genetic operator across a wide range of domains.

5.2 Universality of LMX

Section 5.1 highlighted the connection between LMX and EDAs. This section explores another property of LMX, its theoretical universality (i.e. its ability in theory to express any genetic operator). Interestingly, with a sufficiently expressive class of model, such as Bayesian networks [69, 70], EDAs can approximate any candidate distribution as size of the parent set increases [106]. Not only can LMX sample from distributions represented by an EDA, but it can in principle sample from any conditional probability distribution, making it universal in the space of genetic operators, even with small parent sets. Recent theoretical work has shown how crossover of large neural networks can yield universal approximation of reproduction distributions [61]. LMX also achieves theoretical universal approximation via large neural networks, but by feeding parents directly into the LLM, instead of crossing-over weights. This result follows directly from the universal approximation ability of NNs [18, 36, 43] (note that this property also applies in the single-parent case for mutation-based evolution through LLMs [51]).

This property suggests that the power of LMX is not limited to the randomly-ordered-parent-concatenation-based crossover demonstrated in this paper, but could be used to produce (manually or automatically) crossover behavior optimized for specific tasks, e.g., through prompt-engineering. This ability to achieve arbitrarily complex and diverse reproductive behavior within a single framework gives LMX a distinct advantage over genetic operators that are hand-designed for different tasks: In theory, LMX can represent all such operators (especially if they appear in the dataset used to train the LLM). The generative distributions in the experiments in this paper are limited to ones induced by pre-trained off-the-shelf LLMs, but the underlying universality of the LMX method in general provides further explanation for how it can be an effective generic operator across such a wide range of domains, as was demonstrated in Section 4.

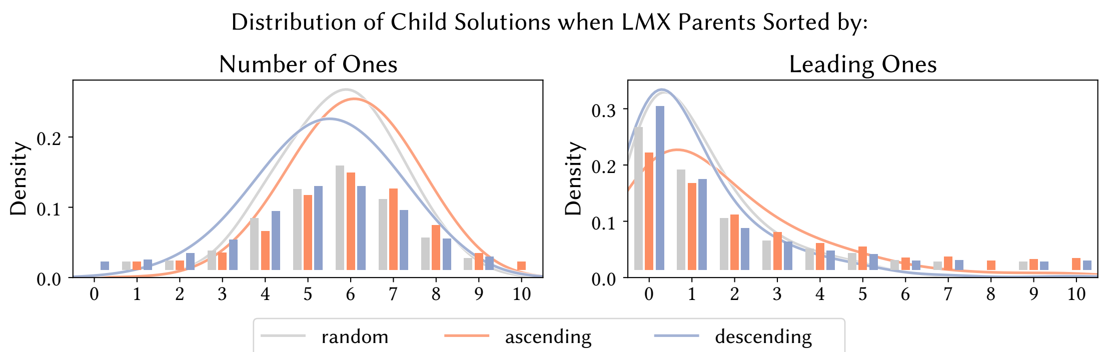


Fig. 15. *Impact of parental sorting on the distribution of offspring produced using the LMX operator.* The same set of parents, represented as bitstrings, was sorted in ascending, descending, or random order based on one of two fitness criteria: number of 'ones' (Left) and leading 'ones' (Right). Kernel Density Estimation (KDE) curves reflect the overall offspring distribution trends, bars represent number of individual offspring. The offspring distribution patterns mirror the parental sorting order, underscoring the influence of parent order on the LMX operator's output. Results are cumulative over 100 experiments with 10 children produced in each experiment.

5.3 Biasing the LMX Operator through Parental Ordering

As an example of how LMX could move beyond its instantiation as an LLM-based EDA, this section investigates the effect of the ordering of parents within the parent prompt. EDAs typically assume an unordered distribution, yet the inherent input ordering in autoregressive models creates a unique opportunity for directing the operator's output. Acting as pattern completion engines, these models provide a pathway to guide the sampling process through directional cues. An ascending input prompt produces ascending output. Consequently, an input arranged in ascending fitness order prompts the model to generate output that follows an ascending fitness trend.

Figure 15 illustrates this biasing technique using the LMX operator in the context of the one-max problem. When randomly generated parents are arranged in ascending order of the 'ones' count, the resulting offspring distribution exhibits a clear skew towards higher fitness (left) and vice versa. The Kernel Density Estimation (KDE) curves clearly represent how offspring distribution is influenced by the sorting order of the parents. The precise counts indicate that for scores of 5 or less from a 10-bit string, the offspring are more likely to originate from parents sorted in descending order than ascending. Conversely, scores of 6 or more tend to come from parents sorted in ascending order vs. descending. Ordering based on the number of leading ones yields a different bias consistent with the ordering (right).

These results are suggestive of the versatility of this biasing technique; a variety of sorting strategies could be employed to cater to specific objectives. For example, including auxiliary helper objectives and rankings such as those used in multiobjectivization [37, 62, 82], genotypic niching [28, 58], novelty [52, 54], or quality-diversity [17, 63] could bias LMX to generate solutions that differ from those previous discovered or that exhibit specific attributes.

Though an intriguing research direction worth exploration, this paper has presented LMX at it's most basic and fundamental – in the experiments in Section 4 all orderings are random.

6 DISCUSSION AND CONCLUSIONS

As a flexible and easy-to-use genetic operator, LMX provides a way for EA practitioners to take advantage of the recent revolution in large language models. The experiments tackle a wide range of potential applications, across equations, plain-text sentences, images, and code, leveraging the wide ecosystem of open-source neural networks as means of generating variation, crossing modalities, and measuring both fitness and diversity.

There is much room for future work. The experiments focused on breadth rather than depth, and it is possible that with further effort LMX could enable state-of-the-art results in e.g. symbolic regression. One important direction is to explore the dependence of LMX's performance on qualities of the underlying LLM; the ability of LMX to suggest relevant variation in a particular domain is likely dependent on the LLM's training data (and its size and how well it was trained). For example, the expectation is that if the type of text chosen for evolution (e.g. code in a very new programming language) is not well-represented in the training distribution of a particular LLM, then LMX that relies on that LLM will likely perform poorly. Another interesting question is whether examples fed into LMX could be chosen more deliberately (e.g. only crossing-over similar individuals to get more nuanced variation); preliminary experiments showed some qualitative effect from applying LMX on individuals with similar embeddings, but require further experimentation to validate. One natural future direction is to explore whether there is benefit from combining the recombination capability of LMX with the mutation operators (either prompt-based or diff-model-based) explored in ELM [51]. Of further interest is the possibility for self-improvement of LMX (as in ELM), through fine-tuning the model on successful examples of variation in a domain. A final intriguing possibility is the use of LMX for interactive evolution, e.g. to interactively evolve sentences. code, or images [7, 85].

While LLMs are computationally expensive, all of the experiments in this paper (with exception of the Python Sodaracer experiment) were conducted either through Google Colab notebooks or on a single GPU; the code to run experiments is surprisingly compact, as the LMX method consists mainly of a simple LLM prompting strategy, and interacting with language and image models has become simple through APIs and libraries such as those provided by HuggingFace. In conclusion, there are likely many creative ways to beneficially combine various models together that this paper leaves unexplored; evolution in general is a powerful and easy-to-implement way to quickly explore such possibilities, and LMX in particular is a promising and simple way of instantiating them.

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A BINARY STRING EXPERIMENTAL DETAILS

The base LLM used for these experiments is the Pythia-deduped 800M model. For the scaling experiments, different parameter sizes were used (as noted in the text). All models are hosted on Huggingface. These Pythia models are trained by EleutherAI for ongoing research [4].

Samples from the LLM were set at a maximum of 150 tokens. For these experiments, rather than using the temperature hyperparameter for controlling LLM sampling, the top-k and top-p hyperparameters are used. Top-k restricts the LLM to output only from the k highest-probability tokens. Top-p further restricts the tokens to be the top tokens that cumulatively take up p of the probability mass. With mild tuning, for all experiments in this section, top-p was set to 0.8 and top-k was set to 30.

The evolutionary algorithm used tournament selection of two with an elitism of 1; the population size was 30.

Figure 4 shows the heritability of LMX in the binary strings domain, and Figure 5 shows how fitness evolves in OneMax using LMX.

A.1 Binary Strings Model Scaling

In this experiment, the number of parents is fixed to 3, and a range of models from the Pythia suite are applied in the same way as in the variation experiment of section 4.1, i.e. to generate variation from randomly-sampled binary strings (although in this experiment they are of length 9 as opposed to length 6). When averaged over 15 randomly-generated parent sets, both the percent of valid offspring and number of novel offspring generally increase with model size (Figure 3).

B SYMBOLIC REGRESSION EXPERIMENTAL DETAILS

The sampling temperature was set to 0.8. All other sampling parameters were defaults. GALACTICA 1.3B was used as the LLM [91].

The initial population had 1000 candidates, and population size was set to 50 thereafter. Any generated offspring that was already in the population was immediately discarded without being evaluated. To prevent stagnation with this relatively small population size, throughout evolution, there was always a 0.05 probability of generating a new candidate directly from the prior set of benchmark expressions (randomly selecting an expression and randomly mapping variables) instead of through LMX. The benchmark expressions are popular benchmarks, whose python representations were copied from the 'deep-symbolic-optimization' GitHub repository (github.com/brendenpetersen/deep-symbolic-optimization/).

Text length for the LLM was capped at 500 tokens. Running 5000 generations took around 100hrs. The vast majority of wall-clock time is spent in the forward pass of the LLM. This could be reduced considerably through batching offspring generation, which is naturally parallelized.

C MODIFYING SENTIMENT EXPERIMENTAL DETAILS

The LLM used in this experiment for LMX is the 1.4 billion parameter Pythia model, hosted on HuggingFace. As in the binary string experiment, for sampling, top-p was set to 0.8 and top-k was set to 30. The max number of tokens generated was set to 128.

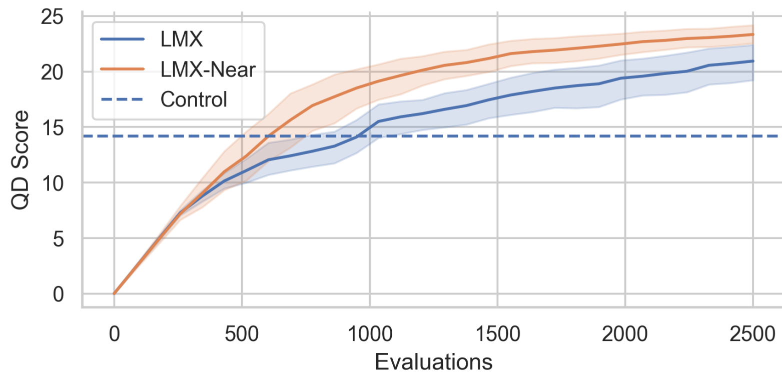

Fig. 16. Modifying Gore Vidal Quote Sentiment. The plot compares LMX-Near, LMX, and the baseline control in increasing the positive sentiment of the quote: “Whenever a friend succeeds, a little something in me dies.” LMX-Near outperforms LMX significantly, and both significantly outperform the control. Example sentences of such runs are shown in appendix section C.1.

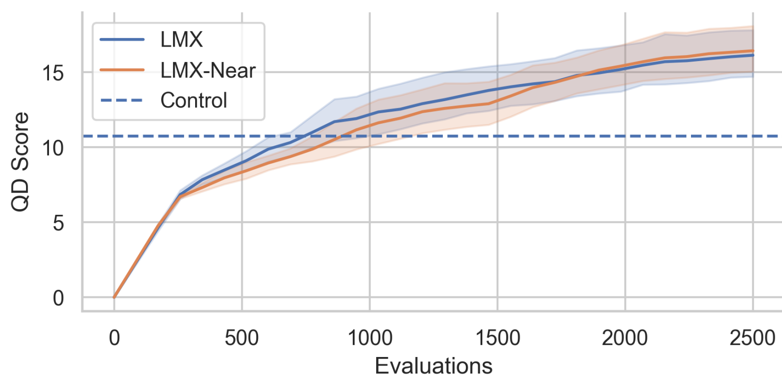

Fig. 17. Modifying Woody Allen Quote Sentiment. The plot compares LMX-Near, LMX, and the baseline control in increasing the positive sentiment of the quote: “Life is divided into the horrible and the miserable.” LMX and LMX-Near do not perform significantly differently, but both significantly outperform the control. Example sentences of such runs are shown in appendix section C.1.

Figure 16 shows fitness plots for the Gore Vidal quote, Figure 10 shows fitness plots for the Homer Simpson quote, and Figure 17 shows fitness plots for the Woody Allen quote. Further examples of evolved behavior are shown in appendix section C.1.

C.1 Additional Positive Sentiment Results

The full Pareto front for one representative run of modifying the Simpsons quote sentiment (from LMX-Near) is shown in Table 1.

For the Gore Vidal quote, “Whenever a friend succeeds, a little something in me dies.” a representative Pareto front (from LMX-Near) is shown in Table 2.

Distance	Positivity	Sentence
0.00	0.02	Kids, you tried your best and you failed miserably. The lesson is, never try.
0.09	0.02	Kids, you tried your best, but you failed miserably. The lesson is, never try.
0.12	0.05	Kids, you tried your best, but you failed miserably. The lesson is, always try.
0.20	0.10	Kids, you tried your best, but you failed. the lesson is, never stop trying.
0.21	0.16	Kids, you always tried your best and you failed. The lesson is, never stop trying.
0.25	0.16	Kids, you tried, tried your best, but you failed. The lesson is, never stop trying.
0.28	0.89	Kids, you tried your best. The lesson is, you always succeed.
0.36	0.90	Kids, you tried your best. The lesson is, success is guaranteed.
0.42	0.94	Kids, you did your best. The lesson is, you never stop trying.
0.46	0.96	Kids, you went above and beyond. The lesson is, never fail, but always try.
0.50	0.96	Kids, you always succeed, The lesson is never fail, but always try, and as long as you keep trying, you will succeed.
0.56	0.96	Kids, you have proven yourself a winner. The lesson, is, never give up, but always try, and as long as you keep trying, you will succeed.
0.61	0.98	Kids, you're the best ever. The lesson is, the best always wins.
0.72	0.99	Kids, you're the best. You're the best, the best. The best.
0.79	0.99	Kids, you are the BEST, the BEST the BEST, the BEST FUTURE!
0.83	0.99	-Kids, this was the BEST DAY OF YOUR LIFE!
0.86	0.99	-Kids, today we're going to have the BEST DAY OF OUR LIFE.
0.89	0.99	-Kids, today we're going to have the BEST DAY OF OUR LIFE!!
0.93	0.99	-Kids, we are so happy to have met you. We love you both!!
0.95	0.99	Kids, we are so happy to have met you! We love you both!!
1.00	0.99	Kids we are so excited that you came into our lives today! Thank you for making our day a little brighter.

Table 1. Full pareto front of a representative run of sentiment modification for the Homer Simpson quote.

Distance	Positivity	Sentence
0.00	0.39	Whenever a friend succeeds, a little something in me dies.
0.26	0.66	Whenever a friend succeeds, the little things in me die.
0.30	0.80	When a friend succeeds, the little things in me die.
0.31	0.89	When a friend succeeds, I die a little.
0.40	0.95	When a friend succeeds, a little thing in me lives.
0.52	0.95	if a friend succeeds, a big thing in me lives.
0.56	0.98	If a friend succeeds, a great thing comes out of me.
0.59	0.98	If a friend succeeds, that's the most awesome thing that's happened to me.
0.63	0.99	If a friend succeeds, I get an exciting feeling in my life, because of them.
0.66	0.99	If a friend succeeds, my friends have the most exciting feeling in my life, because of them.
0.69	0.99	If a friend succeeds, I have the most excitement in my life, because of them.
0.82	0.99	And I'm happy for this friend–I'm happy for this friend.
0.88	0.99	and I'm so happy that I found my new best friend, I'm so happy that I found my new best friend,

Table 2. Full pareto front of a representative run of sentiment modification for the Gore Vidal quote.

For the Woody Allen quote, "Life is divided into the horrible and the miserable", a representative Pareto front (from LMX-Near) is shown in Table 3.

C.2 Evolving towards Negative Sentiment

We also did some initial experiments targeting the negative sentiment class instead of the positive one, i.e. taking positive quotes and turning them negative. As in the experiments in the paper, LMX is able to successfully evolve modifications to quotes that achieve high negativity. However, it often does so by evoking vulgar language or dark situations (e.g. the death of loved ones, or depressive thoughts about hate).

It does often make resigned versions of common inspirational quotes; e.g. one negative version of "Be the change that you wish to see in the world," it produces is "you can't be the change you want to see in the world." From the same run, the most negative sentence on the Pareto front is: "you are the world's worst failure, you have not had good news for the last six months, and you will never find a way to make it up." From the inspirational quote "When the sun is shining

Distance	Positivity	Sentence
0.00	0.01	Life is divided into the horrible and the miserable.
0.20	0.35	Life is, not divided into the horrible and the miserable.
0.24	0.46	Life is, not divided into the horrible or the miserable.
0.30	0.65	For you are the Life, not divided into the horrible, the miserable.
0.34	0.70	You are the Life, not divided into the horrible or the miserable.
0.41	0.75	This is the eternal life, not divided into the horrible or the miserable.
0.46	0.79	You are the eternal life, not divided into the horrible or the miserable.
0.49	0.89	You are the beautiful life, not divided into the horrible or the miserable.
0.51	0.90	You will see the beautiful life, not divided into the horrible or the miserable.
0.53	0.91	You will be the beautiful life, not divided into the horrible or the miserable.
0.73	0.97	Happiness is the way to live.
0.79	0.99	Happiness is the way to live. And I'm very happy with the way that I live.
0.83	0.99	My life is wonderful, I'm very happy with the life.
0.84	0.99	We will live in the glorious happiness. And it is really good, it is really good. And I'm very happy with the life that I have.
0.92	0.99	And I'm very happy with the life that I have. And I can't wait to see the next one.

Table 3. Full pareto front of a representative run of sentiment modification for the Woody Allen quote.

I can do anything; no mountain is too high, no trouble too difficult to overcome," it creates a dreary version: "The earth and the mountains beat me hard, the winds blow heavily, the weather is bitter and cold; I cannot do anything."

While the results are not always pleasant, these preliminary experiments highlight that by using a different classification label (or potentially a different model that recognizes different properties of text altogether), it is possible to use LMX for style-transfer of possibly many other styles.

D IMAGE GENERATION EXPERIMENTAL DETAILS

The image generation experiment used Stable Diffusion v1.4 as the text-to-image model for generating images from evolved prompts; specifically, the 16-bit weight variant (fp16), run from the HuggingFace `diffusers` library.[7] Images were generated at the default 512×512 resolution, and generation was run for 10 diffusion steps per image. While the

[7] https://github.com/huggingface/diffusers

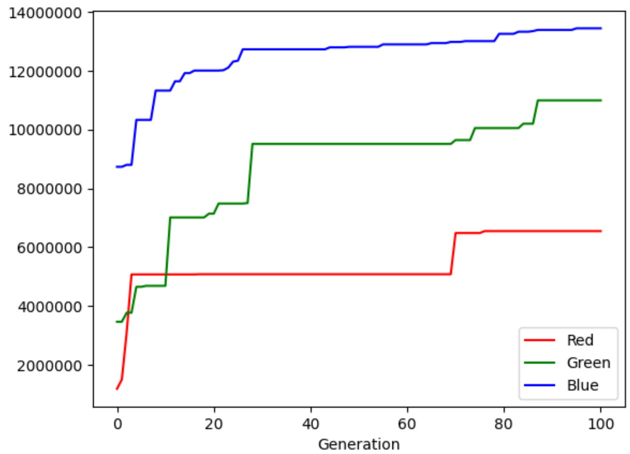

Fig. 18. Image generation progress. Max fitness per generation over a single run using each of the three fitness functions.

default number of steps is normally 50, performance was valued over image fidelty. We left the default NSFW filter enabled, which produces a black image when triggered.

Image fitness functions were computed using 8-bit integer RGB images; the maximum fitness is therefore $512 \cdot 512 \cdot 255 = 66{,}846{,}720$, which would be the fitness of a monochromatic image of the target color.

Pythia-deduped 2.8b was used as the LLM. This is from the same Pythia model series discussed in Appendix A. Up to 75 tokens were sampled from the LLM for each LMX-generated prompt, to stay under Stable Diffusion's limit of 77 tokens in a prompt (Pythia and Stable Diffusion have slightly different tokenizers).

The GA loop used for these experiments was identical to the one from the symbolic regression experiments, but with a smaller population size of 25. The same tournament selection scheme was used, as well as the same 0.05 probability of drawing a new human-written prompt from the initial dataset instead of performing LMX (0.95 probability of generating a new prompt through LMX). Four parents were used as prompts to the LLM to produce each LMX-generated child. The number of parents was not tuned for this problem, but chosen based on the results in Figure 2. The parents were given to the language model almost verbatim, with light prompt engineering. Each parent was placed in a paragraph by itself prefixed by "Prompt: ". The list of parents ended with an open "Prompt:" to request that a child be generated.

On an NVIDIA GeForce RTX 3090, with a population size of 25, each generation took about 2 minutes of wall-clock time. The images in Figure 12 each took a little over 3 hours each to evolve over 100 generations. About 75% of the time was spent in the forward pass of the language model, and 25% in text-to-image generation (everything else was negligible).

E PYTHON SODARACERS EXPERIMENTAL DETAILS

Experiments for the Sodaracers domain were carried out using Salesforce's CodeGen suite of language models [66], using the 350M, 2B, and 6B sizes in their 'mono' variant. The 'mono' models were first pre-trained on natural language, before being fine-tuned on a large dataset of code in many languages, before finally being fine-tuned on a dataset of Python only code. All model sampling was done with top p = 0.95, temperature = 0.85, and with a maximum generation length (in addition to the prompt) of 512 tokens. Evolutionary runs, as described in Section 4.5, took up to 30 hours (at 6B scale) to run on a single Nvidia A100 40GB GPU, while smaller models were significantly quicker. Use of Nvidia's Triton Inference Server has the potential to speed up sampling from these language models by up to an order of magnitude.

The seven Sodaracers used as our seed programs are described in the appendix of Lehman et al. [51]: the square, radial, wheel, runner, galloper, CPPN-Fixed, and CPPN-Mutable programs (CPPN stands for Compositional Pattern-Producing Network [87]).

The Sodaracers were evaluated in a Python simulation of the Sodarace domain [90] written in Box2D (from the Open ELM project [8]). The fitness function was measured as the horizontal distance travelled by an instantiated robot after 1 second of simulation time.

As observed in prior work with few-shot prompting of language models [57], we noticed that success rates (the percentage of generations which resulted in valid Sodaracers) varied dramatically with the order of parent functions in the prompt, sometimes by over 50%. To control for this we either averaged our results over every possible permutation of parents or (in long evolution runs) randomly selected from the set of permutations for each sample.

The main experiments in the paper, described in Section 4.5, prompt the language model with a concatenation of the seed functions, any necessary Python import statements, and the line `def make_walker():` was appended to the end, in order to 'force' the language model to complete a function with this signature.

We also experimented with removing this signature from the end, which produces slightly worse results, particularly in terms of the validation rate. For single-seed prompt mutation, all generations failed to validate, while for LMX with two or three parents the validation rate fell by 15% compared with the main prompt.

In addition, we investigated adding an 'instruction' to the end of the LMX prompt, consisting of a string such as 'Combine the starting programs above to create a new program'. This provides some minimal domain customisation to the language model, and is reminiscent of prior work demonstrating that fine-tuning language models on tasks described as instructions can dramatically improve performance on unseen tasks [78, 100].

Our experiments with instruction prompting demonstrate an intriguing direction for future work with instruction-finetuned language models, which may offer improved quality and diversity of evolved programs or strings if prompted in a way compatible with their training data.

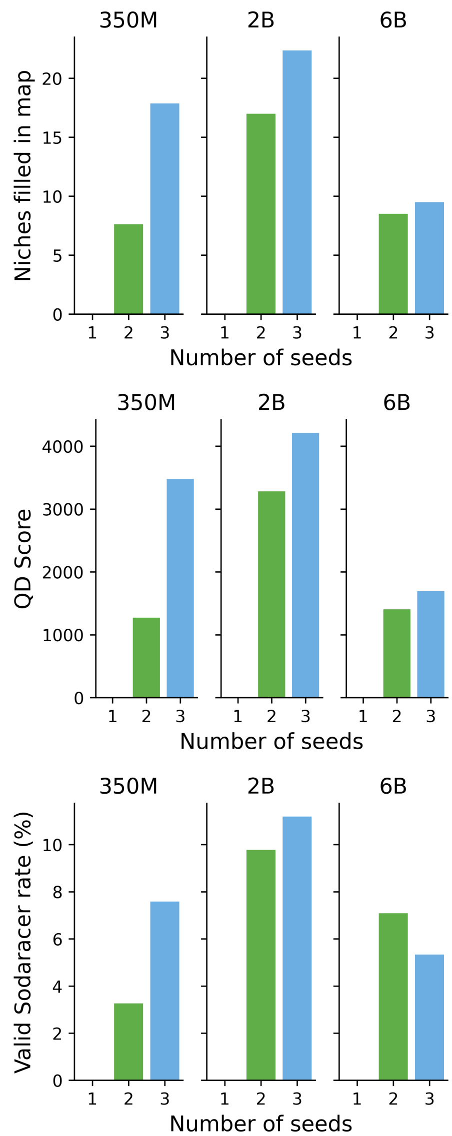

Fig. 19. Results from initial experiments in the Sodaracers domain, using a generic domain-free prompt as described in Appendix E, for varying numbers of parents (seeds) in the language model prompt and across language model scale. This experiment consists of 1000 steps of initialization of Sodaracers from the seeds with no evolution. (top) Number of niches filled in MAP-Elites. (center) Quality-Diversity scores (sum of the fitnesses of all niches in the map) (bottom) Validation rate (%) for the generated Sodaracers. Higher numbers of parents nearly always increases performance in this setting, and the 2B model performs the best.